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ABSTRACT

This document is one of three made up of newsletters from the School Mathematics Study Group's (SMSG) series of newsletters written between 1959 and 1972. This set contains newsletters 1, 4, 6, 11, 24, and 36, covering the history, objectives, organization, and projects undertaken by SMSG; newsletters 13, 25, and 28, discussing the preparation and articulation of content in SMSG tests; newsletter 33, which discusses mathematics for disadvantaged and low-achieving students; and newsletter 38, which states objectives and minimum goals for mathematics education. Related documents are SE 017 486 and SE 017 487. (JP)

SCHOOL
MATHEMATICS
STUDY GROUP

ED 088705

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Newsletter No. 1

March 1959

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TO ERIC AND ORGANIZATIONS OPERATING UNDER AGREEMENTS WITH THE NATIONAL INSTITUTE OF EDUCATION. FURTHER REPRODUCTION OUTSIDE THE ERIC SYSTEM REQUIRES PERMISSION OF THE COPYRIGHT OWNER

To keep the mathematical community informed of its plans and progress, the School Mathematics Study Group will issue a newsletter from time to time. This first Newsletter, *devoted to the history, organization, present activities, and future plans of the School Mathematics Study Group, is being sent to each member of the National Council of Teachers of Mathematics. Those who wish to receive subsequent issues of the* Newsletter *should so indicate, using the form printed on the inside back cover.*

Financial support of the
School Mathematics Study Group
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National Science Foundation

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HISTORY

In the spring of 1958, after consulting with the Presidents of the National Council of Teachers of Mathematics and the Mathematical Association of America, the President of the American Mathematical Society appointed a small committee of educators and university mathematicians to organize a School Mathematics Study Group whose objective would be the improvement of the teaching of mathematics in the schools. Professor E. G. Begle was appointed Director of the Study Group, with headquarters at Yale University. In addition, the organizing committee appointed an Advisory Committee, consisting of college and university mathematicians, high school teachers of mathematics, experts in education, and representatives of science and technology, to work with the director.

The National Science Foundation, with an initial grant of $100,000 in the spring of 1958 and a further grant of $1,200,000 later that year, has provided the financial support for the work of the Study Group.

OBJECTIVES

The world of today demands more mathematical knowledge on the part of more people than the world of yesterday and the world of tomorrow will make still greater demands. Our society leans more and more heavily on science and technology. The number of our citizens skilled in mathematics must be greatly increased; an understanding of the role of mathematics in our society is now a prerequisite for intelligent citizenship. Since no one can predict with certainty his future profession, much less foretell which mathematical skills will be required in the future by a given profession, it is important that mathematics be so taught that students will be able in later life to learn the new mathematical skills which the future will surely demand of many of them.

To achieve this objective in the teaching of school mathematics three things are required. First, we need an improved curriculum which will offer students not only the basic mathematical skills but also a deeper understanding of the basic concepts and structure of mathematics. Second, mathematics programs must attract and train more of those students who are capable of studying mathematics with profit. Finally, all help possible must be provided for teachers who are preparing themselves to teach these challenging and interesting courses.

Each project undertaken by the School Mathematics Study Group is concerned with one or more of these three needs.

ORGANIZATION

Basic policy for the School Mathematics Study Group is set by the Advisory Committee. This Committee meets periodically to review the progress of current projects and decides on the undertaking of new projects. A list of the members of the Advisory Committee is to be found on page 5.

An Executive Committee of the Advisory Committee is available to handle urgent problems between meetings of the Advisory Committee.

Each specific project undertaken by the School Mathematics Study Group is supervised by a Panel, drawn in part from the Advisory Committee. A list of the members of the Panels as currently constituted may be found on pages 6 and 7.

ADVISORY COMMITTEE

A. A. Albert, University of Chicago
F. B. Allen, Lyons Township High School, LaGrange, Illinois
E. G. Begle, Yale University
Lipman Bers, New York University
S. S. Cairns, University of Illinois
G. F. Carrier, Harvard University
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J. H. Hlavaty, Dewitt Clinton High School, New York
P. S. Jones, University of Michigan
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K. O. May, Carleton College
J. R. Mayor, American Association for the Advancement of Science
A. E. Meder, Jr., Rutgers University
F. E. Moise, University of Michigan
P. M. Naghdi, University of California
Richard Pieters, Phillips Academy, Andover, Massachusetts
G. B. Price, University of Kansas
R. E. K. Rourke, Kent School, Kent, Connecticut
A. W. Tucker, Princeton University
Henry Van Engen, University of Wisconsin
A. D. Wallace, Tulane University
E. L. Walters, William Penn Senior High School, York, Pennsylvania
Marie S. Wilcox, Thomas Carr Howe High School, Indianapolis, Indiana
M. W. Zemansky, The College of the City of New York, New York

PANEL ON 7TH AND 8TH GRADES

J. A. Brown, State University Teachers College, Oneonta, New York
Lenore John, University of Chicago
B. W. Jones, University of Colorado
P. S. Jones, University of Michigan
J. R. Mayor, American Association for the Advancement of Science
P. C. Rosenbloom, University of Minnesota
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PANEL ON MONOGRAPHS

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H. S. M. Coxeter, University of Toronto, Toronto, Ontario, Canada
P. R. Halmos, University of Chicago
J. H. Hlavaty, Dewitt Clinton High School, New York
N. Jacobsen, Yale University
H. O. Pollak, Bell Telephone Laboratories
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R. S. Pieters, Phillips Academy, Andover, Massachusetts
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W. W. Sawyer, Wesleyan University
N. E. Steenrod, Princeton University
J. J. Stoker, New York University
Leo Zippin, Queens College, Flushing, New York

PANEL ON TEACHER TRAINING MATERIALS

J. B. Adkins, Phillips Exeter Academy, Exeter, New Hampshire
H. F. Fehr, Columbia University
J. L. Kelley, University of California
L. C. Lay, Pasadena City College
K. O. May, Carleton College
B. E. Meserve, Montclair State Teachers College, Montclair, New Jersey
G. S. Young, University of Michigan
(The Director is *ex officio* a member of each Panel)

RELATIONSHIPS WITH OTHER ORGANIZATIONS

The School Mathematics Study Group is, of course, not the only organization concerned with the improvement of mathematics in our schools. Some of the others are: the Secondary School Curriculum Committee of the National Council of Teachers of Mathematics; the University of Illinois Committee on School Mathematics; and the Commission on Mathematics of the College Entrance Examination Board.

The School Mathematics Study Group is not in competition with any of these organizations. Instead, we believe that we can supplement and extend their work. Liaison with them is provided by the simple expedient of including on our Advisory Committee and on our writing teams representatives of these organizations.

The Federal Government also is vitally interested in the improvement of school mathematics, as is evidenced, for example, by the financial support provided by the National Science Foundation for the School Mathematics Study Group. Even more important is the program of Summer, Academic Year, and In-Service Institutes sponsored by the National Science Foundation, without which any widespread improvement in the teaching of school mathematics would be virtually impossible.

PROJECTS UNDER WAY

EXPERIMENTAL PROJECT—GRADES 7 AND 8

The School Mathematics Study Group believes it particularly important that greater substance and interest be given to the mathematics of grades 7 and 8. One of our major projects now under way is concerned with the curriculum for these two grades. Unlike the curriculum for grades 9 through 12, the curriculum for these two grades has not had the benefit of much careful study in recent years. Our approach has therefore been experimental. About a dozen experimental units have been written and are now being tried out in approximately one hundred classes scattered over the whole country. Some of these experimental units deal with standard topics treated from a new point of view; others present material that is likely to be novel at this level. Our general point of view has been to think of grades 7 and 8 not as the end of elementary school mathematics, but rather as a foundation for the work of the senior high school. We have tried to include in the experimental units a better intuitive basis for the algebra and geometry courses which should follow.

Preliminary reports from classrooms in which these chapters are being tried indicate that students find mathematics from this new point of view very interesting.

In this work we have coöperated very closely, sharing both textbook material and personnel, with a similar project at the University of Maryland.

At a writing session to be held this coming summer, we plan to produce a complete 7th-grade course together with additional supplementary units for the 8th grade, taking into account this year's classroom experience with our present units. These new materials will then be tried out during the academic year 1959–60 and an 8th-grade course will be written during the summer of 1960.

The foregoing experimental units for grades 7 and 8 are now available for inspection. See the order form on the inside back cover.

SAMPLE TEXTBOOKS—GRADES 9–12

A second project now under way is aimed at the production of a series of sample textbooks for grades 9 through 12. The topics discussed in these textbooks will not differ markedly from those included in the present-day high school courses for these grades. However, the organization and presentation of these topics will be different and, we hope, improved. The curriculum which these textbooks will illustrate will stress both concepts and skills. It is hoped that it will provide a better understanding of the nature of both inductive and deductive reasoning and at the same time lead to a better appreciation of mathematical structure.

We do not believe that any single curriculum is to be preferred to the exclusion of all others, and we intend the curriculum embodied in these textbooks to be no more than a sample of the kind of curriculum which we hope to see in our schools.

We foresee a variety of ways in which these textbooks will be used. They will, we hope, provide a model and a source of suggestions for the authors of the classroom textbooks of the future. They will be used in a wide variety of classes to see how new topics in mathematics and new points of view are handled by students of different ability levels and by teachers with different degrees of preparation. We expect that a major use of these textbooks will be in connection with both pre-service and in-service training of teachers, since they will provide concrete illustrations of how an increased emphasis on basic concepts and on mathematical structure can be brought into the classroom.

At a writing session held at Yale University during the summer of 1958, detailed outlines for these textbooks were prepared, after a careful inspection of the publications and recommendations of other organizations interested in high school mathematics. At a second writing session to be held at the University of Colorado during the summer of 1959, we plan to prepare complete preliminary editions of these texts together with accompanying teachers' guides. These will be used the following academic year in a variety of classrooms selected by the School Mathematics Study Group. During the summer of 1960 they will be revised in the light of the experience thus gained and will then be published in noncommercial fashion. They will be kept available for a period of about five years, by which time we hope that they will have served their purpose.

We plan to make the preliminary editions of these texts freely available in the fall of 1959. However, since they do present mathematics from a new point of view, classroom use of them is not recommended until the teacher has become thoroughly familiar with their contents.

The Panel on Sample Textbooks considered the possibility of publishing the outlines prepared last summer. They decided not to do so in view of their tentative nature. Moreover, it was felt that complete texts were needed to contribute effectively to the purposes mentioned above.

MONOGRAPHS

A third project is aimed at the production of a series of short expository monographs on various mathematical subjects. The primary objectives of such monographs are: to disseminate good mathematics at the secondary school level which will supplement the usual high school curriculum; to awaken interest among gifted students; and to present mathematics as a satisfying, meaningful human activity. They are not intended as texts, but rather as supplementary reading material for students, their teachers, and the general educated lay public.

Outstanding mathematicians will write these monographs. In order to be sure that they are understandable and enjoyable by the audience for whom they are intended, preliminary versions will be read by high school students and experienced high school teachers. Their comments, criticisms, and suggestions will be passed on to the authors to form a basis for revision, if necessary.

The monographs will be published as paper-backs by a commercial publisher. The first few will be available during the fall of 1959, and additional ones will appear the following spring.

TEACHER TRAINING MATERIALS

A fourth project, just getting under way, is devoted to the production of textbooks, study guides, etc., for teachers who wish additional training in mathematics. Particular attention will be paid to the production of material suitable for use in Summer and In-Service Institutes, such as those sponsored by the National Science Foundation.

The Panel on Teacher Training Materials will cooperate with the Committee on the Undergraduate Program of the Mathematical Association of America, one of whose concerns is the pre-service training of mathematics teachers.

Two publications in this area are now available from the School Mathematics Study Group. The first of these is a study guide in modern algebra for teachers who wish to improve their professional competence by study either individually or in small groups. The second is a monograph "Some Basic Mathematical Concepts" written by Professor R. D. Luce of Harvard University. See page 18 for a fuller description of these two publications.

TESTING OF NEW TEXT MATERIALS

We believe that new text materials should be thoroughly tested. We do this in two ways. Our experimental units for grades 7 and 8, for example, are being used in approximately 100 classrooms in various parts of the country. After finishing each unit, the teacher fills out a brief questionnaire concerning the ability level of the students, the teachability of the unit, and the need for supplementary material. We plan a similar treatment for our sample textbooks for grades 9 through 12 during the next academic year.

In addition, very careful tests are being carried out for us by the Minnesota National Laboratory for the Improvement of Secondary School Mathematics, a division of the Minnesota State Department of Education. Carefully matched experimental and control classes and teachers use our 7th- and 8th-grade units. Standard achievement tests are used. In addition, new tests are being developed, to measure, for example, attitudes toward and interest in mathematics.

Further information concerning this interesting Laboratory will appear in a future issue of this Newsletter.

FUTURE ACTIVITIES OF SMSG

There are a number of other areas in which the School Mathematics Study Group will carry out studies and, in some cases, undertake projects. Among these are:

I. The use of television, films, and filmstrips both as teaching aids and as teacher training aids. It is hoped that a study of some of the problems in this important and difficult area will get under way this spring.

II. The sample textbooks for grades 9 through 12 mentioned above are aimed at the college-capable student. The School Mathematics Study Group has an equal concern for the less able student, and work on a program and materials for such students will be undertaken in the near future. As a first step, arrangements are being made to try our 9th-grade text with students of intermediate ability, in the hope that this material, taken more slowly and presented more intuitively, will prove both feasible and useful for such students.

III. Text materials for gifted students are badly needed. Some material of this kind will be included, as optional sections, in our textbooks. More will be prepared when the preliminary editions of the textbooks are completed.

IV. A conference on elementary school mathematics, sponsored by the School Mathematics Study Group, was held in Chicago on February 13 and 14, 1959. This conference recommended a comprehensive study of the entire mathematics curriculum from Kindergarten through grade 12. Numerous specific suggestions were made concerning this study and a committee was requested to formulate detailed plans and to present its recommendations to the Advisory Committee of the School Mathematics Study Group.

V. Plans are being made to enlist the aid of social scientists, particularly psychologists, in our work. One outcome of the Chicago conference on elementary school mathematics was an increased awareness on the part of all the participants that the contributions of psychology to the teaching of mathematics are potentially very great.

THE 1958 WRITING SESSION

The outlines of the sample textbooks for grades 9 through 12, and also the experimental units for grades 7 and 8 mentioned above, were produced at a writing session held at Yale University June 23 to July 18, 1958. A list of the participants in this writing session will be found on pages 16 and 17. It will be observed that the participants included both experienced high school teachers and distinguished research mathematicians. Indeed, *it is a fundamental belief of the School Mathematics Study Group that substantial improvement in the school mathematics curriculum can result only from coöperation, on an equal basis, of both these groups.*

The most important outcome of this writing session was proof that such coöperation is possible and productive. Many of the participants came with a certain amount of scepticism on this point. But this soon vanished and was replaced by enthusiasm and a determination to see the work through to a conclusion. It is this coöperation and enthusiasm that leads us to believe that the School Mathematics Study Group can indeed make a substantial contribution to the improvement of the teaching of mathematics in our schools.

PARTICIPANTS IN THE 1958 WRITING SESSION

F. B. Allen, Lyons Township High School, LaGrange, Illinois

E. F. Beckenbach, The RAND Corporation, Santa Monica, California

Emil Berger, Monroe High School, St. Paul, Minnesota

R. H. Bing, University of Wisconsin

J. A. Brown, State University Teachers College, Oneonta, New York

Hope Chipman, University High School, Ann Arbor, Michigan

Mary P. Dolciani, Hunter College

Edwin Douglas, The Taft School, Watertown, Connecticut

E. A. Dudley, North Haven High School, North Haven, Connecticut

Florence Elder, West Hempstead High School, West Hempstead, New York

W. E. Ferguson, Newton High School, Newton, Massachusetts

Joyce D. Fontaine, North Haven High School, North Haven, Connecticut

Esther O. Gassett, Clairmore High School, Clairmore, Oklahoma

E. Glenadine Gibb, Iowa State Teachers College, Cedar Falls, Iowa

R. A. Good, University of Maryland

Martha Hildebrandt, Provisio Township High School, Maywood, Illinois

Lenore John, University High School, University of Chicago

B.W. Jones, University of Colorado

M. L. Keedy, University of Maryland

Marguerite Lehr, Bryn Mawr College

Eunice Lewis, Laboratory High School, University of Oklahoma
M. A. Linton, William Penn Charter School, Philadelphia, Pennsylvania
J. R. Mayor, American Association for the Advancement of Science
K. G. Michaels, North Haven High School, North Haven, Connecticut
E. E. Moise, University of Michigan
E. P. Northrop, University of Chicago
O. J. Peterson, Kansas State Teachers College, Emporia, Kansas
R. S. Pieters, Phillips Academy, Andover, Massachusetts
H. O. Pollak, Bell Telephone Laboratories
G. B. Price, University of Kansas
Persis O. Redgrave, Norwich Free Academy, Norwich, Connecticut
D. E. Richmond, Williams College
C. E. Rickart, Yale University
P. C. Rosenbloom, University of Minnesota
Harry Ruderman, Hunter College High School, New York City
Veryl Schult, Supervisor of Mathematics, Washington, D.C.
Henry Swain, New Trier Township High School, Winnetka, Illinois
A. W. Tucker, Princeton University
H. E. Vaughan, University of Illinois
John Wagner, University of Texas
R. J. Walker, Cornell University
A. D. Wallace, Tulane University
E. L. Walters, William Penn Senior High School, York, Pennsylvania
William Wooton, Verdugo Hills High School, Tujunga, California
J. H. Zant, Oklahoma State University

PUBLICATIONS AVAILABLE

TO ORDER, USE THE FORM ON THE OPPOSITE PAGE AND SEND TO School Mathematics Study Group
Drawer 2502A Yale Station
New Haven, Connecticut

STUDY GUIDE IN MODERN ALGEBRA

The purpose of this Study Guide is to provide assistance to teachers who wish to improve their professional competence in modern algebra by study either individually or in small groups. It contains a list of basic topics and ideas which should be part of the mathematical equipment of teachers of introductory (9th grade) algebra. There is a short bibliography and, for each of the basic topics and ideas, detailed references to this bibliography.

Price: FREE

SOME BASIC MATHEMATICAL CONCEPTS,

by R. D. LUCE

An exposition of elementary set theory, together with illustrations of the use of set concepts in various parts of mathematics.

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Fourteen experimental units for these grades, together with a teacher's guide for each unit and answers to the problems.

Price: $2.00

SMSG NEWSLETTER

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Newsletter No. 4

March 1960

"PERMISSION TO REPRODUCE THIS COPYRIGHTED MATERIAL HAS BEEN GRANTED BY

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This Newsletter *may be considered a progress report to the mathematicians of this country, on the work of the School Mathematics Study Group.*

The aim of this Study Group—better mathematics in our schools—is spelled out in more detail in the following pages, as is also the program being followed in furtherance of this aim. Here we wish merely to point out that a very sizeable number of mathematicians have devoted a great deal of thought and effort to developing materials pointing some new directions which could lead to substantial improvements in school mathematics.

The task, however, is an enormous one and cannot be fully achieved by any one group. To assure progress, colleges and universities will have to make an even greater contribution than they are now making.

An encouragingly large number of high school teachers would like to do a better job, and they should be provided with all the help they need. More, and improved, summer and in-service institutes are necessary, as well as more summer school programs designed for teachers and emphasizing subject matter. The pre-service training of teachers should also be improved to allow us to look forward to a continuing improvement in the education of our children.

Individual mathematicians can contribute by designing and teaching subject matter courses for teacher training and also by acting as consultants to groups of teachers trying to introduce improved courses.

If the enthusiasm of all those participating in this Study Group can be reflected in the work which must be done, and can only be done, by our colleges and universities, then we have a good chance of achieving success in this task which is so important to the welfare of our country.

Financial Support of the
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has been provided by the
National Science Foundation

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HISTORY

In the spring of 1958, after consulting with the Presidents of the National Council of Teachers of Mathematics and the Mathematical Association of America, the President of the American Mathematical Society appointed a small committee of educators and university mathematicians to organize a School Mathematics Study Group whose objective would be the improvement of the teaching of mathematics in the schools. Professor E. G. Begle was appointed Director of the Study Group, with headquarters at Yale University. In addition, the organizing committee appointed an Advisory Committee, consisting of college and university mathematicians, high school teachers of mathematics, experts in education, and representatives of science and technology, to work with the director.

The National Science Foundation, through a series of grants, has provided very substantial financial support for the work of the Study Group.

OBJECTIVES

The world of today demands more mathematical knowledge on the part of more people than the world of yesterday and the world of tomorrow will make still greater demands. Our society leans more and more heavily on science and technology. The number of our citizens skilled in mathematics must be greatly increased; and understanding of the role of mathematics in our society is now a prerequisite for intelligent citizenship. Since no one can predict with certainty his future profession, much less foretell which mathematical skills will be required in the future by a given profession, it is important that mathematics be so taught that students will be able in later life to learn the new mathematical skills which the future will surely demand of many of them.

To achieve this objective in the teaching of school mathematics three things are required. First, we need an improved curriculum which will offer students not only the basic mathematical skills but also a deeper understanding of the basic concepts and structure of mathematics. Second, mathematics programs must attract and train more of those students who are capable of studying mathematics with profit. Finally, all help possible must be provided for teachers who are preparing themselves to teach these challenging and interesting courses.

Each project undertaken by the School Mathematics Study Group is concerned with one or more of these three needs.

ORGANIZATION

Basic Policy for the School Mathematics Study Group is set by the Advisory Committee which meets periodically to review the progress of current projects and to decide on the undertaking of new projects. Each specific project is under the supervision of a Panel, drawn in part from the Advisory Committee. The membership of the Advisory Committee and of the various Panels may be found on pages 12 through 17.

RELATIONSHIPS WITH OTHER ORGANIZATIONS

The School Mathematics Study Group is, of course, not the only organization concerned with the improvement of mathematics in our schools. Some of the others are: the Secondary School Curriculum Committee of the National Council of Teachers of Mathematics; the University of Illinois Committee on School Mathematics; the Commission on Mathematics of the College Entrance Examination Board, and the University of Maryland Mathematics Project. Liaison with them is provided by the simple expedient of including on the Advisory Committee, and in the work of each Panel, representatives of these organizations.

The Federal Government also is vitally interested in the improvement of school mathematics, as is evidenced, for example, by the financial support provided by the National Science Foundation for the School Mathematics Study Group. Even more important is the program of Summer, Academic Year, and In-Service Institutes sponsored by the National Science Foundation, without which any widespread improvement in the teaching of school mathematics would be virtually impossible.

PROJECTS UNDER WAY

I. MATHEMATICS FOR GRADES 7 AND 8

The School Mathematics Study Group believes it particularly important that greater substance and interest be given to the mathematics of grades 7 and 8. Our general point of view has been to think of grades 7 and 8 not as the end of elementary school mathematics, but rather as a foundation for the work of the senior high school. The curriculum for these grades should include a sound intuitive basis for the algebra and geometry courses to follow.

To provide a concrete illustration of this kind of curriculum, textbooks for these two years were prepared at a writing session at the University of Michigan in the summer of 1959. Brief descriptions of these texts may be found on page 19.

Accompanying each of these texts is a commentary for the teacher. These commentaries include not only the usual materials (discussion of teaching problems, solutions for the exercises, etc.), but also discussions and deeper expositions of the mathematics.

These texts are now being used in twelve Experimental Centers, involving about 100 teachers and 8,500 students. Preliminary reports from these Centers are most encouraging and in particular teachers and students alike find mathematics from this point of view much more interesting.

The teachers in these Centers report regularly on their classroom experiences. Revisions of the texts, taking into account these reports, will be prepared in the summer of 1960.

These two texts are based on a series of individual units which were written in the summer of 1958, in a writing session at Yale University, and tried out in classes during 1958–59. These units are described on page 19. They have proved useful as supplements to conventional texts for these grades and have been made generally available for this purpose.

II. MATHEMATICS FOR GRADES 9 THROUGH 12

This project is devoted to the production of a series of sample textbooks for grades 9 through 12. For the most part the topics discussed in these textbooks do not differ markedly from those included in the present-day high school courses for these grades. However, the organization and presentation of these topics is different. Important mathematical skills and facts are stressed, but equal attention is paid to the basic concepts and mathematical structures which give meaning to these skills and provide a logical framework for these facts.

We do not believe that any single curriculum is to be preferred to the exclusion of all others, and we intend the curriculum exemplified in these textbooks to be no more than a sample of the kind of curriculum which we hope to see in our schools.

We foresee a variety of ways in which these textbooks will be used. They will, we hope, provide a model and a source of suggestions for the authors of classroom textbooks of the future. A major use of these texts will be in connection with pre-service and in-service training of teachers, since they will provide concrete illustrations of how an increased emphasis on basic concepts and on mathematical structure can be brought into the classroom. Finally, they will provide a stop-gap till such texts become available through the usual channels (in this connection see page 10).

Preliminary versions of these texts were prepared at a writing session held at the University of Colorado in the summer of 1959, using detailed outlines which had been prepared at the Yale writing session in the summer of 1958. Brief descriptions of these texts may be found on pages 20–22.

As with the texts for grades 7 and 8, a commentary for the teacher accompanies each text.

These texts are now being used in a total of 37 experimental centers involving approximately 260 teachers and 18,000 students. As in the case of the project described above, preliminary reports from these centers are most encouraging.

The teachers using these texts report on their classroom experiences and, on the basis of these reports, the texts will be revised during the summer of 1960.

III. MONOGRAPHS

A third project is aimed at the production of a series of short expository monographs on various mathematical subjects. The primary objectives of such monographs are: to disseminate good mathematics at the secondary school level which will supplement the usual high school curriculum; to awaken interest among gifted students; and to present mathematics as a satisfying, meaningful human activity. They are not intended as texts, but rather as supplementary reading material for students, their teachers, and the general educated lay public.

Outstanding mathematicians will write these monographs. In order to be sure that they are understandable and enjoyable by the audience for whom they are intended, preliminary versions will be read by high school students and experienced high school teachers. Their comments, criticisms, and suggestions will be passed on to the authors to form a basis for revision, if necessary.

These monographs will be published as paperbacks by a commercial publisher. First drafts of a half-dozen manuscripts had been prepared at the time of publication of this Newsletter.

IV. TEACHER TRAINING MATERIALS

Practically all recommendations for improved secondary school mathematics curricula that have been seriously proposed, either by SMSG or by others, involve aspects of mathematics which have not, in the past, been included in the normal subject matter training of secondary school teachers. This project is devoted to the production of materials specifically for teachers who wish the additional training in mathematics needed to teach an improved curriculum. Particular attention is paid to materials suitable for use in summer and in-service institutes, such as those sponsored by the National Science Foundation.

Two series of publications are under way. The first is a series of study guides for teachers who wish to improve their professional competence by study either individually or in small groups. One of these, in algebra, is already available (see page 18), and others on analysis, geometry, logic, number theory, and probability are in preparation.

The second series consists of brief expositions of various topics in mathematics designed explicitly for in-service teachers. Two of these are already available (see page 18), and another, for teachers of the SMSG First Course in Algebra, will be available at the beginning of the summer.

V. MATHEMATICS FOR NON-COLLEGE-BOUND STUDENTS

The sample textbooks mentioned above for grades 9 through 12 were written explicitly for college-capable students. This particular project is devoted to the construction of an improved curriculum for less able students. As a first step it will test the hypothesis that such students can learn the kind of mathematics contained in the SMSG 9th- and 10th-grade texts provided that the material is presented in a less formal fashion and with more concrete illustrations, and provided that the students are allowed to proceed at their own pace. Appropriate revisions of these texts will be made in the summer of 1960 in order to carry out this test. A similar test will be conducted with the SMSG texts for grades 7 and 8.

VI. ELEMENTARY SCHOOL MATHEMATICS

In this project SMSG will undertake a critical study of the elementary school mathematics curriculum from the point of view of: increased emphasis on concepts and mathematical principles; the grade placement of topics in arithmetic; the introduction of new topics, particularly from geometry; and supplementary topics for the better students, for example from number theory.

A start on this will be made in the summer of 1960 in a writing session which will prepare experimental materials for grades 4–6.

OTHER ACTIVITIES

Text materials for gifted students are badly needed. Some material of this kind is included, as optional sections in the textbooks mentioned above and others will be published separately.

In another direction, an experiment is now under way which will test the feasibility of a correspondence course for gifted students. This, if successful, will be one method for providing for the gifted students located in schools too small to offer special sections or courses.

Many students develop in school a negative attitude toward mathematics and hence are lost to science and technology. The SMSG sample textbooks are now being studied, by a group including both mathematicians and social scientists, to see how they affect attitudes toward mathematics.

CLASSROOM USE OF SMSG TEXTS

GENERAL POLICY

It was pointed out above (page 6) that the major purposes of the SMSG textbooks are, first, to provide a concrete illustration of the kind of mathematics curriculum we would like to see in our schools, and secondly to provide materials useful in teacher training. However, numerous inquiries have already reached SMSG Headquarters concerning the availability of these texts for classroom use until similar texts become available through the normal commercial channels. The purpose of this section is to answer these inquiries.

When an SMSG text is decided to be satisfactory for classroom use, it will be published in non-commercial fashion and made generally available. However, it will be kept available for no more than five years, in the belief that by the end of this period it will have served its major purposes.

General procedure with an SMSG text is to restrict classroom use of the preliminary version to the SMSG Experimental Centers. The text and the teachers' commentary are then revised, taking into account this classroom experience. In some cases a text may be heavily dependent on a preceding text and a second year of classroom use in Experimental Centers followed by a second revision may be necessary.

ACADEMIC YEAR 1960–61

For the academic year 1960–61 SMSG proposes to make available revised versions of the texts for grades 7, 9, 10, and 12. These revised texts will be furnished with much more durable covers than the ones now used on the preliminary versions. It is believed that with these covers the classroom life of a text will be at least two years. The price of a new text, including the more durable cover, will not exceed $2.50.

It is not known at present if revised versions of the 8th- and 11th-grade texts can be made available for the coming academic year. These two texts are more heavily dependent on preceding SMSG texts than is true for the others.

It would be of considerable assistance if those who are contemplating the use of SMSG texts next academic year would so inform SMSG Headquarters, indicating the number of copies of each text that might be ordered. This will make it possible to forward information on exact prices and instructions for ordering as soon as they are determined.

The following comments may be of some help to school authorities thinking of adopting SMSG texts.

It should be kept in mind that most secondary school teachers, through no fault of their own, were not provided in their pre-service training with the mathematics which the use of these texts requires. Consequently, most teachers will need some help, in the form of additional training in mathematics, when teaching these texts for the first time. However, experience in the SMSG Experimental Centers suggests that an in-service training program, taught by a subject matter specialist, either before or during the first year's use of the texts, will be quite satisfactory in answer to this problem.

Evidence from the SMSG 7th- and 8th-grade Centers indicates that when a teacher teaches an SMSG course for the second time, the amount of in-service assistance needed and the extra time needed for preparation are both drastically reduced, and in many cases disappear entirely.

An important source of the needed additional mathematical training is the program of summer and in-service institutes sponsored by the National Science Foundation. A number of the NSF summer institutes in 1960 will concentrate heavily on SMSG courses, and many others will undoubtedly use SMSG materials as supplements to their regular courses.

However, SMSG cannot itself provide any direct assistance to school systems wishing to use SMSG texts. In fact, a basic principle in America is that education is locally controlled and is independent of the Federal Government. SMSG, which receives all its financial support from the Federal Government through the National Science Foundation, therefore wishes to do nothing which might be interpreted as an attempt to influence this local control of education. SMSG will confine its activities to the preparation and testing of improved texts. The decision to adopt these texts, and the implementation of the decision, is entirely up to the local school systems.

A final comment is that there already is considerable evidence that adoption of SMSG texts is an almost irreversible process. Most teachers are very reluctant to return to conventional textbooks after once using SMSG texts.

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(The Director is *ex officio* a member of each Panel)

PARTICIPANTS IN THE WRITING SESSIONS

Below are listed the names of all those who participated in one or more of the three SMSG writing sessions. It will be observed that the participants included both experienced high school teachers and distinguished research mathematicians. Indeed, *it is a fundamental belief of the School Mathematics Study Group that substantial improvement in the school mathematics curriculum can result only from coöperation, on an equal basis, of both these groups.*

The mere existence of the sample textbooks for grades 7–12 is ample proof that such coöperation is not only possible, but also productive.

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TO ORDER, USE THE FORM ON THE INSIDE BACK COVER AND SEND TO School Mathematics Study Group
Box 2029 Yale Station
New Haven, Connecticut

STUDY GUIDE IN MODERN ALGEBRA

The purpose of this Study Guide is to provide assistance to teachers who wish to improve their professional competence in modern algebra by study either individually or in small groups. It contains a list of basic topics and ideas which should be part of the mathematical equipment of teachers of introductory (9th grade) algebra. There is a short bibliography and, for each of the basic topics and ideas, detailed references to this bibliography.

Price: FREE

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An exposition of elementary set theory, together with illustrations of the use of set concepts in various parts of mathematics.

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VOLUME 2. EUCLIDEAN GEOMETRY BASED ON RULER AND PROTRACTOR AXIOMS (revised edition)

by C. W. CURTIS, P. H. DAUS, and R. J. WALKER

A study, written for high school teachers, of the approach to Euclidean Geometry used in the SMSG tenth-grade textbook.

Price: $1 00

NOTE: This revised edition differs from the preliminary edition published in the spring of 1959 in that Chapter 6 of the preliminary edition was completely re-written and a new Chapter 8 was added. Owners of the preliminary edition may obtain, at no cost, an addendum containing this new material.

JUNIOR HIGH SCHOOL MATHEMATICS UNITS

These are the materials for grades 7 and 8 prepared by SMSG in the summer of 1958 and tried in Experimental Centers in the academic year 1958–59. They have been grouped into three volumes. Volume I, "Number Systems" discusses numeration, natural numbers, factoring and primes, tests for divisibility, rational numbers, and abstract mathematical systems. Volume II, "Geometry," discusses non-metric geometry, informal geometry, and measurement. Volume III, "Applications," contains a unit entitled "What is Mathematics and Why You Need to Know It," and other units on the lever, statistics, and chance.

Each volume is accompanied by a Commentary for Teachers. In these are included digests of the reports from the teachers who taught these Units in their classes. These Units are now available for classroom use. The prices for these are: VOLUME I, $ .75, VOLUME II, $ .50, VOLUME III, $ .25.

The price of a Commentary for Teachers is the same as the price of the volume which it accompanies.

MATHEMATICS FOR JUNIOR HIGH SCHOOL

VOLUMES I AND II.

Emphasized in these texts are the following important ideas of junior high school mathematics: structure of arithmetic from an algebraic viewpoint; the real number system as a progressing development; metric and non-metric relations in geometry. These ideas are constantly associated with their applications. Time is given to the topics of measurement and elementary statistics. Careful attention is paid to the appreciation of abstract concepts, the role of definition, development of precise vocabulary and thought, experimentation, and proof. Materials are chosen with the intent to capture the fascinating features of mathematics, creation and discovery, rather than just utility alone. The texts provide adequate preparation for the remaining work in the SMSG sequence.

Price: VOL. I $3.50
VOL. II $2.00

MATHEMATICS FOR HIGH SCHOOL FIRST COURSE IN ALGEBRA

This text emphasizes the structure of algebra. The study of algebra is based on the exploration of the behavior of numbers. Careful attention is paid to the language of the subject. While most of the material which traditionally makes up a first course in algebra is woven into the text, this material appears in an entirely different light since it is organized and motivated by fundamental structure considerations. This preliminary edition does not presuppose a study of the SMSG Junior High School mathematics program.

Price: $4.25

GEOMETRY

This text is designed for the one-year introductory course in geometry which is usually taught in the tenth grade. The book is devoted mainly to plane geometry, with some chapters on solid geometry, and an introduction to analytic geometry. Solid geometry is introduced very early in the text and is used as a foil to develop the students' space perception. There are a considerable number of formal proofs in solid geometry, in some cases integrated with the plane geometry and in other cases in special chapters. The book is written in the belief that the traditional content of Euclidean geometry amply deserves the prominent place which it now holds in the high school curriculum. Changes were made only when the need for them appeared to be compelling. The basic scheme in the postulates is that of George D. Birkhoff. It is assumed that the students are familiar with the number line, and numbers are used freely for measuring both distances and angles. Geometry is connected with algebra at every reasonable opportunity. Statements of definitions and theorems are exact without making exactitude a substitute for intuitive insight. The text is written in the belief that intuition and logic should move forward hand in hand.

Price: $3.75

INTERMEDIATE MATHEMATICS

This text is predicated on the philosophy that a student's interest in mathematics grows as he becomes aware of difficulties, overcomes them, and, on the basis of the mastery thus gained, becomes aware of greater difficulties. It is written with the feeling that it is in the eleventh grade where able students reach higher ground in a mathematical sense. Careful attention has been taken to give the student some insight into the nature of mathematical thought as well as to prepare him to perform certain manipulations with facility. Included in the text are chapters devoted to trigonometry, vectors, logarithms, mathematical induction, and complex numbers written with a much higher degree of sophistication than usually appears at this level. The text continues in the vein of the ninth-grade material, in that structure is emphasized at all times. For the benefit of students who have not previously studied algebra from the point of view of structure, an introductory chapter is provided which reviews elementary algebra from this point of view.

Price: $4.75

ELEMENTARY FUNCTIONS

Again, as in the previous texts, there may be a considerable amount of over-lap with previous SMSG books. This will continue to be so as long as students begin the sequence at varying grade levels. The text is designed for use in the first half of the twelfth grade but by judicious use of supplementary material it could serve as a basis for a longer course. The central theme is a study of functions: polynomial, exponential, logarithmic, and trigonometric, with emphasis on practical applications wherever possible. The introduction of a simple but geometrically meaningful method for handling areas, tangents, and maximum-minimum problems furnishes the student with a good intuitive background for a later course in calculus.

Price: $2.75

INTRODUCTION TO MATRIX ALGEBRA

This text is designed for the last half of the 12th grade. It is devoted to a study of matrices, including applications to solutions of systems of linear equations and to geometry. At the same time careful attention is devoted to algebraic structure, but not to the point where a barren presentation results. Mathematics is introduced which is new to the student and the structure is developed as the text proceeds. It is the intent of the text to put the student close to the frontiers of mathematics and to provide striking examples of patterns that arise in the most varied circumstances. It provides an effective language and some dynamic concepts that will prove useful in many college courses. A special set of "Research Exercises" is appended in the hope that some students may be introduced to real mathematical research. The commentary for teachers will be announced in the next NEWSLETTER.

Price: $1.00

If you are not now on our mailing list but wish to receive further issues of this NEWSLETTER, please request, by means of a post card, that your name be added to the mailing list.

ORDER FORM

All SMSG publications are priced at cost. No reductions can be offered for bulk orders.

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*For prices of individual volumes, see p. 19.

†This is the price for both the student and teacher volumes. These are not sold separately.

Address to which above order should be sent.

Remittance must accompany each order. Please make checks payable to Yale University, and mail to

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Newsletter No. 6

March 1961

DO NOT USE FOR ORDERING
See Current Newsletters,
No. 11 and later, for
available publications.

This Newsletter may be considered a progress report to the mathematicians of this country, on the work of the School Mathematics Study Group.

The aim of this Study Group—better mathematics in our schools—is spelled out in more detail in the following pages, as is also the program being followed in furtherance of this aim. Here we wish merely to point out that a very sizeable number of mathematicians have devoted a great deal of thought and effort to developing materials pointing some new directions which could lead to substantial improvements in school mathematics.

The task, however, is an enormous one and cannot be fully achieved by any one group. To assure progress, colleges and universities will have to make an even greater contribution than they are now making.

An encouragingly large number of high school teachers would like to do a better job, and they should be provided with all the help they need. More, and improved, summer and in-service institutes are necessary, as well as more summer school programs designed for teachers and emphasizing subject matter. The pre-service training of teachers should also be improved to allow us to look forward to a continuing improvement in the education of our children.

Individual mathematicians can contribute by designing and teaching subject matter courses for teacher training and also by acting as consultants to groups of teachers trying to introduce improved courses.

If the enthusiasm of all those participating in this Study Group can be reflected in the work which must be done, and can only be done, by our colleges and universities, then we have a good chance of achieving success in this task which is so important to the welfare of our country.

Financial Support of the
School Mathematics Study Group
has been provided by the
National Science Foundation

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Printed in the U.S.A.

HISTORY

In the spring of 1958, after consulting with the Presidents of the National Council of Teachers of Mathematics and the Mathematical Association of America, the President of the American Mathematical Society appointed a small committee of educators and university mathematicians to organize a School Mathematics Study Group whose objective would be the improvement of the teaching of mathematics in the schools. Professor E. G. Begle was appointed Director of the Study Group, with headquarters at Yale University. In addition, the organizing committee appointed an Advisory Committee, consisting of college and university mathematicians, high school teachers of mathematics, experts in education, and representatives of science and technology, to work with the director.

The National Science Foundation, through a series of grants, has provided very substantial financial support for the work of the Study Group.

OBJECTIVES

The world of today demands more mathematical knowledge on the part of more people than the world of yesterday and the world of tomorrow will make still greater demands. Our society leans more and more heavily on science and technology. The number of our citizens skilled in mathematics must be greatly increased; and understanding of the role of mathematics in our society is now a prerequisite for intelligent citizenship. Since no one can predict with certainty his future profession, much less foretell which mathematical skills will be required in the future by a given profession, it is important that mathematics be so taught that students will be able in later life to learn the new mathematical skills which the future will surely demand of many of them.

To achieve this objective in the teaching of school mathematics three things are required. First, we need an improved curriculum which will offer students not only the basic mathematical skills but also a deeper understanding of the basic concepts and structure of mathematics. Second, mathematics programs must attract and train more of those students who are capable of studying mathematics with profit. Finally, all help possible must be provided for teachers who are preparing themselves to teach these challenging and interesting courses.

Each project undertaken by the School Mathematics Study Group is concerned with one or more of these three needs.

ORGANIZATION

Basic Policy for the School Mathematics Study Group is set by the Advisory Committee which meets periodically to review the progress of current projects and to decide on the undertaking of new projects. Each specific project is under the supervision of a Panel, drawn in part from the Advisory Committee. The membership of the Advisory Committee and of the various Panels may be found on pages 13 through 14.

PROJECTS

I. MATHEMATICS FOR GRADES 7 AND 8

The School Mathematics Study Group believes it particularly important that greater substance and interest be given to the mathematics of grades 7 and 8. Our general point of view has been to think of these grades not just as the end of elementary school mathematics but also as a foundation for the work of the senior high school. The curriculum for these grades should include a sound intuitive basis for the algebra and geometry courses to follow.

To provide a concrete illustration of this kind of curriculum, textbooks for these two years have been prepared. Descriptions of these texts may be found on page 20.

Accompanying each of these texts is a commentary for the teacher. These commentaries include not only the usual materials (discussion of teaching problems, solutions for the exercises, etc.) but also discussions and deeper expositions of the mathematics.

These texts evolved from a series of experimental junior high school units which were prepared at the first SMSG writing session at Yale University in the summer of 1958. In the following summer a preliminary version of a 7th grade text and a collection of 8th grade units, including much of the material in these experimental units was prepared. In the summer of 1960 a revised version of the 7th grade text and a preliminary version of an 8th grade text were prepared. A revision of the 8th grade text will be carried out in the summer of 1961.

During the intervening academic years, these materials were used in a number of experimental centers, involving approximately 100 teachers and 8,000 students each year. Feedback from the teachers was used extensively each summer in the revision of these materials.

II. MATHEMATICS FOR GRADES 9 THROUGH 12

This project is devoted to the production of a series of sample textbooks for grades 9 through 12. For the most part the topics discussed in these textbooks do not differ markedly from those included in the present-day high school courses for these grades. However, the organization and presentation of these topics is different. Important mathematical skills and facts are stressed, but equal attention is paid to the basic concepts and mathematical structures which give meaning to these skills and provide a logical framework for these facts.

Preliminary versions of these texts were prepared at a writing session held at the University of Colorado in the summer of 1959, using detailed outlines which had been prepared at the Yale writing session in the summer of 1958.

As with the texts for grades 7 and 8, a commentary for the teacher accompanies each text.

These texts were used in a total of 37 experimental centers involving approximately 260 teachers and 18,000 students. As in the case of the project described above, the teachers using these texts reported on their classroom experiences and, on the basis of these reports, the texts were revised during the summer of 1960 at a writing session at Stanford University. Brief description of these texts may be found on pages 22–27.

During the summer of 1961 a preliminary version of another geometry text will be prepared. This text will emphasize analytic geometry, and will be tried out in a number of experimental centers during 1961–62.

III. MONOGRAPHS

A third project is aimed at the production of a series of short expository monographs on various mathematical subjects. The primary objectives of such monographs are: to disseminate good mathematics at the secondary school level which will supplement the usual high school curriculum; to awaken interest among gifted students; and to present mathematics as a satisfying, meaningful human activity. They are not intended as texts, but rather as supplementary reading material for students, their teachers, and the general educated lay public.

Outstanding mathematicians will write these monographs. In order to be sure that they are understandable and enjoyable by the audience for whom they are intended, preliminary versions will be read by high school students and experienced high school teachers. Their comments, criticisms, and suggestions will be passed on to the authors to form a basis for revision, if necessary.

These monographs will be published as paperbacks by a commercial publisher. The first six will be ready late in the spring of 1961, at which time a special issue of this NEWSLETTER will include information on prices and instructions for ordering them.

The first six monographs are:

Numbers: Rational and Irrational, by IVAN NIVEN.

What is Calculus About?, by W. W. SAWYER.

An Introduction to Inequalities, by EDWIN BECKENBACH and RICHARD BELLMAN.

Geometric Inequalities, by NICHOLAS D. KAZARINOFF.

The Contest Problem Book. Problems from the Annual High-School Contests of the Mathematical Association of America, compiled by CHARLES T. SALKIND.

The Lore of Large Numbers, by PHILIP J. DAVIS.

IV. TEACHER TRAINING MATERIALS

Practically all recommendations for improved secondary school mathematics curricula that have been seriously proposed, either by SMSG or by others, involve aspects of mathematics which have not, in the past, been included in the normal subject matter training of secondary school teachers. This project is devoted to the production of materials specifically for teachers who wish the additional training in mathematics needed to teach an improved curriculum. Particular attention is paid to materials suitable for use in summer and in-service institutes, such as those sponsored by the National Science Foundation.

Two series of publications are under way. The first is a series of study guides for teachers who wish to improve their professional competence by study either individually or in small groups. A study guide in algebra is currently being revised and others on analysis, geometry, logic, number theory, and probability are in preparation.

The second series consists of expositions of various topics in mathematics designed explicitly for in-service teachers. Five of these are already available (see page 19), and others will be available in the near future.

V. MATHEMATICS FOR LESS ABLE STUDENTS

The sample textbooks mentioned previously for grades 9 through 12 were written explicitly for college-capable students. This particular project is devoted to the construction of an improved curriculum for slower students. This work is under the guidance of the Panel on Underdeveloped Mathematical Talent. As a first step it will test the hypothesis that such students can learn the kind of mathematics contained in the SMSG 9th and 10th grade texts provided that the material is presented in a less formal fashion and with more concrete illustrations, and provided that the students are allowed to proceed at their own pace. Appropriate revisions of these texts were made in the summer of 1960 in order to carry out this test. A similar test is being conducted with the SMSG texts for grades 7 and 8.

Pending definite information on the feasibility of these materials the books are not available for general distribution at this time.

VI. ELEMENTARY SCHOOL MATHEMATICS

In this project SMSG will undertake a critical study of the elementary school mathematics curriculum from the point of view of: increased emphasis on concepts and mathematical principles; the grade placement of topics in arithmetic; the introduction of new topics, particularly from geometry; and supplementary topics for the better students, for example from number theory.

A start on this was made in the summer of 1960 in a writing session at Stanford University. A complete course for grade 4 and selected units for grades 5 and 6 were prepared. A commentary for the teacher accompanies each unit.

These units are now being used in a total of 27 experimental centers involving approximately 12,000 students with 150 fourth grade, 110 fifth grade, and 110 sixth grade teachers.

These teachers report their classroom experiences and, on the basis of these reports, the units will be revised during the summer of 1961. The work for grades 5 and 6 will be completed, and during the academic year, 1961–62, a complete course of study for mathematics grades 4, 5, and 6 will be classroom tested.

A brief description of the current preliminary elementary school mathematics will be found on pages 30–31.

OTHER ACTIVITIES

Text materials for gifted students are badly needed. Some material of this kind is included, as optional sections in the textbooks mentioned above and others have been published separately. See page 28. Detailed plans are being considered for a considerable expansion of the program of publication of brief supplementary materials for students of varied ability levels.

In another direction, an experiment is now under way which will test the feasibility of a correspondence course for gifted students. This, if successful, will be one method for providing for the gifted students located in schools too small to offer special sections or courses.

Many students develop in school a negative attitude toward mathematics and hence are lost to science and technology. The SMSG sample textbooks are now being studied, by a group including both mathematicians and social scientists, to see how they affect attitudes toward mathematics.

The Educational Testing Service is currently conducting a study of the performance of students using SMSG texts as compared to those using conventional texts.

A careful study of teaching machines, programmed learning, etc., will be carried out with special reference to the SMSG text materials.

GENERAL INFORMATION ON USES OF SMSG TEXTS

There are a number of uses to which the SMSG texts can be put. They will, it is believed, provide a model and a source of suggestions for the authors of future classroom textbooks. A second major use of these texts will be in connection with the pre-service and in-service training of teachers, since they provide concrete illustrations of how an increased emphasis on basic concepts and on mathematical structure can be brought into the classroom. Finally, they will provide a stop-gap until similar texts become available through the usual channels.

A number of inquiries have already been received concerning the possibility of incorporating SMSG text materials in contemplated commercial textbooks. The advisory committee of SMSG has approved the following statement of policy on this matter.

SMSG has prepared a series of sample texts designed to illustrate, in a concrete fashion, the kind of curriculum which the members of SMSG believe should be taught in the schools of this country. It is not intended that the SMSG texts be considered as the only suitable ones for such a curriculum. Indeed a variety of texts of the same general nature is not only possible but also highly desirable. It is expected that these will become available through the usual commercial channels in the near future.

A major purpose of an SMSG text is to serve as a model and as a course of suggestions and ideas for the authors of this variety of texts. Textbook writers should feel free to use SMSG texts in this way and to adapt, expand, and improve them for their own purposes. SMSG would appreciate being given proper credit in such cases, but at the same time it should be made clear that no endorsement by SMSG is implied.

Pending the availability of commercial texts along the general lines advocated by SMSG, each SMSG text will be kept available as long as it is needed. There will be an annual review, and when it is determined that a text is no longer needed, it will no longer be kept available. Suitable warning of such a decision will be given.

The SMSG texts are copyrighted and may not be reproduced without permission. Normally, permission will be granted for the reproduction of selections from the SMSG texts for purposes consistent with those of SMSG.

As to classroom use of SMSG texts, the general procedure has been to restrict such use of preliminary versions to the SMSG Experimental Centers. The text and teacher's commentary are revised, taking into account this classroom experience, as many times as seems necessary.

During the course of development of an SMSG text, the preliminary versions are usually made available for information purposes and are sold at cost.

When it is decided that a particular SMSG text is in a satisfactory state from the point of view of the three purposes listed above, the text then is made available in a non-commercial fashion.

At the present time final revisions of the following texts have been completed: Mathematics for Junior High School, Vol. I, First Course in Algebra, Geometry, Intermediate Mathematics, Elementary Functions, and Introduction to Matrix Algebra. It is expected that a final revision of Mathematics for Junior High School, Vol. II will be completed in the summer of 1961.

Descriptions of these texts will be found on pages 20 through 27.

AVAILABILITY OF SMSG TEXTS FOR CLASSROOM USE

ACADEMIC YEAR 1960–61

For the academic year 1960–61 the above mentioned texts were made available for classroom use. A total of slightly over 100,000 texts were ordered for this purpose. This large total, far exceeding expectations, so taxed the capacities of the printers producing these texts that it proved impossible to do more than supply these classroom orders. Now, however, we have been able to reprint these texts in moderate numbers and they are now available in limited quantities for inspection, and for use in in-service training of teachers, including use in summer institutes. An order blank at the end of this booklet should be used when ordering any of these texts.

ACADEMIC YEAR 1961–62

Arrangements are now being concluded with the Yale University Press for the printing and distribution of SMSG texts. Details of this arrangement together with instructions for ordering these texts for classroom use for 1961–62 will be contained in a later edition of this NEWSLETTER, which will appear in April 1961.

It is not expected that the cost of the SMSG texts for classroom use in 1961–62 will differ substantially from the prices listed at the end of this NEWSLETTER.

NECESSITY OF IN-SERVICE PREPARATION OF MATHEMATICS TEACHERS— EXPERIENCE IN SMSG EXPERIMENTAL CENTERS

It should be kept in mind that most secondary school teachers, through no fault of their own, were not provided in their pre-service training with the mathematics which the use of these texts requires. Consequently, most teachers will need some help, in the form of additional training in mathematics, when teaching these texts for the first time. However, experience in the SMSG Experimental Centers suggests that an in-service training program, taught by a subject matter specialist, either before or during the first year's use of the texts, will be quite satisfactory in answer to this problem.

Evidence from the experimental centers indicates that when a teacher teaches an SMSG course for the second time, the amount of in-service assistance needed and the extra time needed for preparation are both drastically reduced, and in many cases disappear entirely. A detailed discussion of SMSG's experience with in-service assistance to teachers is found in NEWSLETTER No. 5.

An important source of the needed additional mathematical training is the program of summer and in-service institutes sponsored by the National Science Foundation. A number of the NSF Summer Institutes in 1961 will concentrate heavily on SMSG courses, and many others will undoubtedly use SMSG materials as supplements to their regular courses.

However, SMSG cannot itself provide any direct assistance to school systems wishing to use SMSG texts. In fact, a basic principle in America is that education is locally controlled and is independent of the Federal Government. SMSG, which receives all its financial support from the Federal Government through the National Science Foundation, therefore wishes to do nothing which might be interpreted as an attempt to influence this local control of education. SMSG will confine its activities to the preparation and testing of improved texts. The decision to adopt these texts, and the implementation of the decision, is entirely up to the local school systems.

A final comment is that there already is considerable evidence that adoption of SMSG texts is an almost irreversible process. Most teachers are very reluctant to return to conventional textbooks after once using SMSG texts.

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(The Director is *ex officio* a member of each Panel)

PARTICIPANTS IN THE WRITING SESSIONS

Below are listed the names of all those who participated in one or more of the SMSG writing sessions. It will be observed that the participants included both experienced high school teachers and distinguished research mathematicians. Indeed, *it is a fundamental belief of the School Mathematics Study Group that substantial improvement in the school mathematics curriculum can result only from coöperation, on an equal basis, of both these groups.*

The mere existence of the sample textbooks for grades 7–12 is ample proof that such coöperation is not only possible, but also productive.

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NOTE: The institutions listed above are those with which the participants were associated during the writing sessions.

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TO ORDER, USE THE FORM ON THE INSIDE BACK COVER AND
SEND TO School Mathematics Study Group
Box 2029 Yale Station
New Haven, Connecticut

STUDIES IN MATHEMATICS

VOLUME 1. SOME BASIC MATHEMATICAL CONCEPTS

by R. D. LUCE

An exposition of elementary set theory, together with illustrations of the use of set concepts in various parts of mathematics.

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by VINCENT H. HAAG

An explanation of the approach to algebra found in the SMSG text "First Course in Algebra" emphasizing the foundations of the subject and the structural properties of elementary algebraic systems.

Price: $1.00

VOLUME 4. GEOMETRY

by B. V. KUTUZOV

A translation of a Russian text for teachers. Longer than other volumes in this series (576 pages), it will probably be used mostly as a source of supplementary material.

Price: $5.00

VOLUME 5. CONCEPTS OF INFORMAL GEOMETRY

by Richard D. Anderson

A study of basic ideas, concepts, and points-of-view of geometry, intended primarily for junior high school teachers.

Price: $1.00

MATHEMATICS FOR JUNIOR HIGH SCHOOL

VOLUME I (revised edition)
AND
VOLUME II (preliminary edition)

Traditional mathematics courses for grades 7 and 8 include a review of the operations with whole numbers, fractions, and decimals. Percent is introduced, usually in terms of the three cases of percents, each of which is treated separately after various manipulations with percents, including fractional and decimal equivalents of percents. The traditional courses also have rather extensive treatments of percent applications such as commission, simple interest, discount, and insurance. A study of measurement has had an important place, but again much of this is a review of work done in earlier grades and little or none of it new from a mathematical point of view.

While the new SMSG courses provide for review of the fundamentals of arithmetic, this review has been placed in a new setting with emphasis on number systems. Number systems are treated from an algebraic viewpoint not only to deepen the student's understanding of arithmetic but also to prepare him for the algebra which is to come. The work on fractions is introduced by defining a fraction as a numeral for the rational number $\frac{a}{b}$ such that $b(\frac{a}{b}) = a, b \neq 0$. The grade 8 text starts with an informal treatment of coördinates and equations, and includes a brief introductory chapter on probability. Some of the probability problems were written by biologists associated with the Biological Sciences Curriculum Study, and in a chapter on the lever an attempt has been made to use language consistent with that of the Physical Science Study Committee physics course. Percent applications have a place in the new courses, as do other social applications, for example through governmental statistics in the chapter on graphs and in probability.

The new courses give more than one-third of the time to geometry, which is a very considerable change in emphasis from the traditional. Geometric ideas are introduced, first of all, from a non-metric point of view and then, after a careful treatment of measurement, students are led gradually to a study of properties of triangles and other geometric figures, plane and solid, through an informal deductive approach. Although there is no attempt to give a system of postulates for the geometry, properties are identified on an intuitive or inductive basis and then these properties are used to draw conclusions about, or to prove, other properties. In the chapter on drawings and constructions, instruments in addition to the classical ones are introduced and the student is also provided with experience in sketching figures, especially three-dimensional figures. A grade 8 chapter on non-metric geometry which comes just before the study of the measurement of volumes and surface areas is, in its topological approach, one of the greatest innovations.

VOLUME I: CHAPTER HEADINGS

1. What Is Mathematics?
2. Numeration
3. Whole Numbers
4. Non-Metric Geometry
5. Factoring and Primes
6. The Rational Number System
7. Measurement
8. Area, Volume, Weight, and Time
9. Ratios, Percents, and Decimals
10. Parallels, Parallelograms, Triangles, and Right Prisms
11. Circles
12. Mathematical Systems
13. Statistics and Graphs
14. Mathematics at Work in Science

Prices:	VOLUME I. Student	$2.50
	VOLUME I. Commentary	$2.50

VOLUME II. CHAPTER HEADINGS

1. Rational Numbers and Coördinates
2. Equations
3. Scientific Notation, Decimals, and the Metric System
4. Drawings and Constructions
5. Symmetry, Congruence and the Pythagorean Property
6. Real Numbers
7. Permutations and Selections
8. Probability
9. Similar Triangles and Variation
10. Non-Metric Geometry
11 Volumes and Surface Areas
12. The Sphere
13. Relative Error

Prices:	VOLUME II. Student	$2.50
	VOLUME II. Commentary	$2.50

MATHEMATICS FOR HIGH SCHOOL

FIRST COURSE IN ALGEBRA (revised edition)

The SMSG ninth grade course is based upon structure properties of the real number system. This development of algebra is interesting, meaningful, and mathematically sound. It helps bring out the nature of mathematics and strengthens the student's algebraic techniques by relating them to basic ideas. Definitions and properties are carefully formulated and there is some work with simple proofs. The reading material, which is an important part of the course, is designed to help the student discover ideas. The number line and the simpler language of sets are used to help express the ideas. Inequalities are treated along with equations.

As its title suggests, the SMSG ninth grade course covers essentially the same material as does a conventional first year algebra text. It teaches the student how to perform the fundamental operations with real numbers and with variables and how to do the usual algebraic manipulations, including factoring, powers and roots, and work with polynomials and fractional expressions. It shows how to solve equations up through quadratic equations in one variable and linear equations in two variables. Graphs of linear and quadratic functions are treated. There is much experience in solving word problems.

CHAPTER HEADINGS

1. Sets and the Number Line
2. Numerals and Variables
3. Sentences and Properties of Operations
4. Open Sentences and English Phrases
5. The Real Numbers
6. Properties of Addition
7. Properties of Multiplication
8. Properties of Order
9. Subtraction and Division for Real Numbers
10. Factors and Exponents
11. Radicals
12. Polynomial and Rational Expressions
13. Truth Sets of Open Sentences
14. Graphs of Open Sentences in Two Variables
15. Systems of Equations and Inequalities
16. Quadratic Polynomials
17. Functions

Prices:	First Course in Algebra. Student	$2.50
	First Course in Algebra. Commentary	$2.50

GEOMETRY (revised edition)

The SMSG geometry text differs from conventional ones in content, postulational scheme, and manner of treatment.

1. No artificial distinction is made between plane and solid geometry, and a considerable amount of the latter is included. Also, an introduction to analytic plane geometry is provided.

2. The postulate system is a modification of Birkhoff's, and is complete. Real numbers are used freely throughout the text, both in the theory and in problems.

3. Accuracy in the statement and use of postulates, definitions, and theorems is emphasized.

On the other hand, the text is still basically a treatment of synthetic Euclidean geometry, covering the usual topics: congruence, similarity, parallelism and perpendicularity, area, circles, and construction with straight-edge and compass. There is a main sequence of proved theorems, some minor stated theorems with proofs left as exercises for the students, and a long list of "originals." The basic postulates, definitions, and theorems are motivated by appeal to intuition, and many practical and computational problems are given.

CHAPTER HEADINGS

1. Common Sense and Organized Knowledge
2. Sets, Real Numbers and Lines
3. Lines, Planes and Separation
4. Angles and Triangles
5. Congruences
6. A Closer Look at Proof
7. Geometric Inequalities
8. Perpendicular Lines and Planes in Space
9. Parallel Lines in a Plane
10. Parallels in Space
11. Areas of Polygonal Regions
12. Similarity
13. Circles and Spheres
14. Characterization of Sets. Constructions
15. Areas of Circles and Sectors
16. Volumes of Solids
17. Plane Coördinate Geometry

Prices:	Geometry. Student	$2.50
	Geometry. Commentary	$2.50

INTERMEDIATE MATHEMATICS (revised edition)

The SMSG eleventh grade text differs from traditional texts in the following important ways:

1. The SMSG text makes much greater demands on the student's *ability to learn by reading carefully worded expositions.* The writers believe that the development of this ability is essential for success in college mathematics.
2. The study of *number systems* is stressed as the basis for all understanding of both elementary and advanced mathematics.
3. The idea that algebra is a *logical structure* built on a relatively small number of fundamental principles is emphasized throughout the text.
4. Presentations which lead the student to make certain pre-determined *"discoveries"* are used where appropriate.
5. *Proof* is emphasized throughout in order that the student may gain some idea of the nature of a valid mathematical argument.
6. The *function concept* is developed spirally throughout the text.

7. *Coördinate geometry* is introduced earlier than usual and is used as a tool in the development of subsequent sections, notably those on trigonometry.
8. The presentation of logarithms reflects contemporary usage which requires more understanding of logarithms and exponential functions and relatively less emphasis on logarithmic computations.
9. The treatment of trigonometry emphasizes identities, equations, and graphs of the trigonometric functions more than the computations required in the solution of triangles.
10. *Vectors* are developed as a mathematical system and are applied to the solution of a wide variety of problems.

The writers hope that through the studying of this text the student will acquire some ability to handle unforeseen and unforeseeable problems.

The SMSG eleventh grade text is similar to conventional texts in these respects.

1. The text begins with a review and extension of the basic skills of first year algebra. This review is included in the initial study of number systems.
2. The content is essentially the same as that found in conventional courses in trigonometry and college algebra.
3. Practical applications are given about the same amount of attention as in conventional texts. It was not possible to increase the number of applications appreciably without making unwarranted assumptions about the student's understanding of related fields.
4. The exposition makes use of many illustrative examples and drawings.
5. There is an abundant supply of exercises which have been carefully graded as to difficulty.

CHAPTER HEADINGS

1. Number Systems
2. An Introduction to Coördinate Geometry in the Plane
3. The Function Concept and the Linear Function
4. Quadratic Functions and Quadratic Equations
5. Complex Number System
6. Equations of the First and Second Degree in Two Variables
7. Systems of Equations in Two Variables
8. Systems of First Degree Equations in Three Variables
9. Logarithms and Exponents
10. Introduction to Trigonometry
11. The System of Vectors
12. Polar Form of Complex Numbers
13. Sequences and Series
14. Permutations, Combinations, and the Binomial Theorem
15. Algebraic Structures

Prices:		
Prices:	Intermediate Mathematics. Student	\$2.50
	Intermediate Mathematics. Commentary	\$2.50

ELEMENTARY FUNCTIONS (revised edition)

The subject matter of *Elementary Functions* is basically conventional. It includes such topics from the theory of equations as the remainder and factor theorems and the usual methods for finding rational roots. The student will find the laws of exponents and logarithms and the rules for changing the base. The chapter on circular functions contains the familiar addition and subtraction formulas and their consequences, identities and equations, and inverse trigonometric functions. An appendix treats the solution of triangles. Emphasis is laid on graphs. However, each of these topics is treated with some novelty and in a new spirit.

Elementary Functions applies the concept of mapping to polynomial, exponential, logarithmic, and circular functions. Effective use is made of the ideas of composition and inversion. The treatment of tangents is intuitive, elementary, and rigorous. It permits the introduction of Newton's method, and applications to maximum-minimum problems and to graphing. The treatment prepares for calculus without trespassing upon it. The explanation of exponentials and logarithms is novel and unusually clear and thorough. Trigonometry is freshly developed in line with the mapping idea. The style is informal. Explanations are full and concrete, and they convey the spirit of mathematical thinking.

CHAPTER HEADINGS

1. Functions
2. Polynomial Functions
3. Tangents to Graphs of Polynomial Functions
4. Exponential and Logarithmic Functions
5. Circular Functions

Prices:

Elementary Functions. Student	$2.00
Elementary Functions. Commentary	$2.00

INTRODUCTION TO MATRIX ALGEBRA (revised edition)

This text is designed for the last half of the 12th grade. It is devoted to a study of matrices, including applications to solutions of systems of linear equations and to geometry. At the same time careful attention is devoted to algebraic structure, but not to the point where a barren presentation results. Mathematics is introduced which is new to the student and the structure is developed as the text proceeds. It is the intent of the text to put the student close to the frontiers of mathematics and to provide striking examples of patterns that arise in the most varied circumstances. It provides an effective language and some dynamic concepts that will prove useful in many college courses. A special set of "Research Exercises" is appended in the hope that some students may be introduced to real mathematical research.

CHAPTER HEADINGS

1. Matrix Operations
2. The Algebra of 2 × 2 Matrices
3. Matrices and Linear Systems
4. Representation of Column Matrices as Geometric Vectors
5. Transformations of the Plane

Prices:

Introduction to Matrix Algebra. Student	$1.50
Introduction to Matrix Algebra. Commentary	$1.50

SUPPLEMENTARY MATERIALS

MATHEMATICS FOR THE JUNIOR HIGH SCHOOL SUPPLEMENTARY UNITS (revised edition)

CHAPTER HEADINGS

1. Sets
2. Special Figures in Projective Geometry
3. Repeating Decimals and Tests for Divisibility
4. Open and Closed Paths
5. Finite Differences
6. Recent Information on Primes
7. Games

Prices:	Mathematics for Junior High School Supplementary Units. Student	$ .75
	Mathematics for Junior High School Supplementary Units. Commentary	$ .75

ESSAYS IN NUMBER THEORY

These supplements were written for students who are especially good in mathematics and who have a lively interest in the subject. The author's aim in (1) and (2) is to lead the reader to discover for himself some interesting results and to experience the thrill of mathematical discovery. The others are more expository in nature, but they contain exercises to clarify the material and to give the reader a chance to work with the concepts which are introduced.

VOLUME I: CHAPTER HEADINGS

1. Prime Numbers
2. Congruence
3. The Fundamental Theorem of Arithmetic

Price: $ .25

VOLUME II: CHAPTER HEADINGS

1. Arithmetic Functions I
2. Arithmetic Functions II
3. Arithmetic Functions III
4. The Euclidean Algorithm and Linear Diophantine Equations
5. The Gaussian Integers
6. Fermat's Method of Infinite Descent
7. Approximation of Irrationals by Rationals
8. A New Field

Price: $ .50

MATHEMATICS FOR HIGH SCHOOL INTERMEDIATE MATHEMATICS (PART I) SUPPLEMENTARY UNIT I

(THE DEVELOPMENT OF THE REAL NUMBER SYSTEM) (revised edition)

This is a revision of Chapter I "The Real Number System," INTERMEDIATE MATHEMATICS (preliminary edition), Part I.

Price: $ .75

CONFERENCE REPORTS

REPORT OF A CONFERENCE ON ELEMENTARY SCHOOL MATHEMATICS

This publication includes abstracts and résumés of the presentation, discussions, and recommendations of a conference on elementary school mathematics held on February 13 and 14, 1959. The participants included college and university mathematicians, high school teachers, educational experts with special interest in arithmetic, supervisors, elementary school teachers, psychologists, and representatives of scientific and government organization having an interest in mathematics.

Price: $ .50

REPORT OF AN ORIENTATION CONFERENCE FOR SMSG EXPERIMENTAL CENTERS

This publication includes abstracts of the presentations and discussions of a conference on mathematics for grades 7–12 held on September 19, 1959. The conference was called so that the teachers and consultants in the experimental centers for 1959–60 could discuss the aims, objectives, and content of the SMSG books for grades 7–12. The participants included a representative sampling of the text authors.

Price: $1.00

REPORT OF AN ORIENTATION CONFERENCE FOR SMSG ELEMENTARY SCHOOL EXPERIMENTAL CENTERS

This publication includes abstracts of the presentations and discussions of a conference on elementary school mathematics held on September 23–24, 1960. The conference was called so that the teachers and consultants in the elementary school experimental centers for 1960–61 could discuss the aims, objectives, and content of the SMSG materials for grades 4, 5, and 6. These participants included a representative sampling of the text authors.

Price: $1.00

MATHEMATICS FOR THE ELEMENTARY SCHOOL

The nature of these materials was described on page 7. Grade placement for the units listed is described in a preface to the volume in which *all* of these units have been collected. These materials are not yet ready for general classroom use and are being made available at this time to inform the mathematical community of the present state of the SMSG project on elementary mathematics. The two volumes, pupil's text and teacher's commentary, are available ONLY as a single unit. They cannot be purchased separately.

UNIT HEADINGS

1. Concepts of Sets
2. Numeration
3. Nature and Properties of Addition and Subtraction
4. Techniques of Addition and Subtraction
5. Sets of Points

5M. Sets of Points

6. Recognition of Common Figures
7. Nature and Properties of Multiplication and Division
8. Techniques of Multiplication and Division
9. Developing the Concept of Fractional Numbers
10. Linear Measurement
11. Factors, Primes, and Common Denominators
12. Properties and Techniques of Addition and Subtraction of Fractional Numbers

13. Side and Angle Relationships of Triangle
14. Measurement of Angles
15. Extending Systems of Numeration
17. Area
19. Multiplication and Division of Fractional Numbers
21. Area of Rectangular Regions
24. Introducing Exponents

Price: $10.00

Books listed on the order blank and described in pages 19 through 30 are the only books available at this time. Supplies of earlier versions of these materials have been exhausted.

In ordering books please use the order blank enclosed in this NEWSLETTER or specify the exact title in full when ordering.

As indicated on page 11 the current supply of materials is limited. Please do not submit orders for CLASSROOM quantities for the academic year 1961–62 at this time. Texts for grades 7 through 12 will be available at approximately the same price for classroom use through the Yale University Press. The April issue of the SMSG NEWSLETTER (No. 7) will contain full particulars.

If you are not now on our mailing list but wish to receive further issues of this NEWSLETTER, please request, by means of a post card, that your name be added to the mailing list.

SCHOOL MATHEMATICS STUDY GROUP
SCHOOL OF EDUCATION—CEDAR HALL
Stanford University
Stanford, California

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SCHOOL
MATHEMATICS
STUDY GROUP

Newsletter No. 11

March 1962

CONTENTS

The Future of SMSG, 3

Bylaws, 4

Membership: Advisory Board and Panels, 7

New Projects, 10

New Publications, 14

Distribution of SMSG Publications, 21

Order Form, center

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THE FUTURE OF SMSG

When the School Mathematics Study Group came into being on March 1, 1958 it was not clear how long it would take to accomplish its objectives. Undoubtedly many of those who participated in the creation of SMSG felt that a brief period of intensive effort would be sufficient.

A few years of experience, however, were enough to make it clear that the general kinds of activities undertaken by SMSG, in particular the close collaboration between classroom teachers and research mathematicians, ought to be continued indefinitely. The completion of several major projects gives evidence of the value of such collaboration; in the course of completing them the need for several new projects developed.

For this reason a conference was held in February 1961 to consider ways of continuing the work which SMSG had begun. The sixty participants represented all parts of the country and all parts of the mathematical profession, as well as various degrees of interest in mathematics education, ranging from full-time participation in curriculum projects to passive but interested observation from the sidelines. The main recommendation resulting from this conference was that SMSG should continue in existence, carrying on with its present programs and, in addition, broadening its scope to provide leadership in all areas of school mathematics education.

In addition, it was recommended that the Advisory Committee, which sets the basic policy for the School Mathematics Study Group, be given a more formal structure and that, in particular, some members of the Advisory Committee be named by the Conference Board of the Mathematical Sciences, which represents all the major mathematical organizations in the United States.

These recommendations were accepted and in October, 1961, the Advisory Committee (now renamed the Advisory Board) adopted a set of bylaws embodying these recommendations. These bylaws follow.

BYLAWS

I. PURPOSES

The primary purpose of the SMSG is to foster research and development in the teaching of school mathematics. This research will consist, in part, of a continual review of the mathematics curricula in the schools, as an aid in the selection and design of promising experiments. It will also consist in part of an analysis of the results of experimental teaching, as an aid in judging whether the objectives of various programs are being achieved. But the work of the SMSG should consist primarily in the development of courses, teaching materials and teaching methods.

A great variety of these are needed. In the first place, the range of ability among the students who ought to be learning mathematics is so wide that special provisions need to be made, so that the students at various ability levels can be taught in appropriate styles and at appropriate paces. Moreover, there should be bold experiments, with courses differing sharply from present practice in their style, or their content, or both; if experimentation is limited to programs whose desirability is obvious and whose feasibility is predictable, then some of the best opportunities are likely to be missed. Sources of other suggestions for needed activities and projects are: (a) the declaration of aims and purposes of the SMSG, July, 1958; (b) the actual activities and productions of the SMSG, 1958-1961; (c) the Recommendations for the Conference on Future Responsibilities for School Mathematics, February 1961.

It is a part of SMSG's task, in cooperation with the several mathematics organizations, to encourage exploration of the hypotheses underlying mathematics education. Care should be taken to give attention to the needs of other disciplines interested in the mathematical preparation of students. It is also part of the SMSG's task to publicize its own work and make this work available to people who might use it. It should be understood, however, that the basic job of the SMSG is not to defend any orthodoxy, old or new, by passing judgment on questions of educational policy, but rather to make contributions to the data on which such judgments must be based. In this spirit, efforts should be made to help teacher-education institutions to offer programs which will enable their graduates to teach the new courses in the schools. The SMSG should also explore the possibility of promoting the developments of in-service education of teachers and supervisors as a continuing and necessary part of their professional lives.

The success of the SMSG's enterprise depends on the full participation of mathematicians from the colleges and universities and classroom teachers from the schools. It is a fundamental policy that the work of the SMSG be collaborative in this sense.

II. ORGANIZATION

The executive officer of the SMSG shall be the Director. There shall also be an Advisory Board and an Executive Committee.

The first Advisory Board of the SMSG shall consist of

(1) The Director, *ex officio*,

(2) Eight members of the SMSG Advisory Committee (as constituted in the Spring of 1961), to serve for two years,

(3) Eight members of the same body, to serve for one year,

(4) Four members to be elected by the same body in the Fall of 1961, to serve for three years, and

(5) Four members to be elected by the Conference Board of the Mathematical Sciences, to serve for three years

Thereafter, the term of each member shall be three years, beginning on September 1 and ending on August 31; and no member shall serve two consecutive terms.

Each year, four new members shall be appointed by the Conference Board of the Mathematical Sciences and four shall be elected by the Advisory Board. Vacancies due to uncompleted terms shall be filled by the Executive Committee.

The Advisory Board shall elect (1) its own chairman, who shall also be the chairman of the Executive Committee, and (2) three other members of the Executive Committee. All of these officers shall serve for one-year terms, beginning on September 1 and ending on August 31.

The Executive Committee shall consist of its four elected members, together with the Director, *ex officio*. If the directorship becomes vacant, a new Director shall be appointed by the Executive Committee.

The chairman of the Advisory Board shall preside at all meetings of the Advisory Board and the Executive Committee. Each year he shall appoint a nominating committee, which shall make nominations for all of the annual elections by the Advisory Board. At the meeting at which these elections are held, nominations from the floor shall be permitted.

Meetings of the Advisory Board and the Executive Committee shall be called by the Director. The Director shall also appoint such ad hoc committees, panels and writing teams as may be needed, and prepare reports on SMSG activities on request of the Advisory Board.

Amendments to the Bylaws may be made by the Advisory Board.

MEMBERSHIP
—ADVISORY BOARD
—PANELS

ADVISORY BOARD 1961-62

Terms Expire September 1, 1962

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Terms Expire September 1, 1963

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Chairman of the Advisory Board
September 1, 1961 - August 31, 1962
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Terms Expire September 1, 1964

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*Members of the Executive Committee
**Appointed by the Conference Board of the Mathematical Sciences

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R. A. Rosenbaum, Wesleyan University
Henry Swain, New Trier Township High School, Winnetka, Illinois
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W. G. Chinn, Portola Junior High School, San Francisco
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E. E. Moise, Harvard University
H. O. Pollak, Bell Telephone Laboratories
D. W. Taylor, Yale University

PANEL ON SMALL PUBLICATIONS

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M. P. Bridgess, Roxbury Latin School, West Roxbury, Massachusetts
J. M. Calloway, Kalamazoo College, Kalamazoo, Michigan
R. J. Clark, St. Paul's School, Concord, New Hampshire
Roy Dubisch, University of Washington
T. H. Hill, Oklahoma City Schools
K. S. Kalman, Lincoln High School, Philadelphia, Pennsylvania
Augusta L. Schurrer, Iowa State Teachers College, Cedar Falls, Iowa
Henry W. Syer, Kent School, Kent, Connecticut

(The Director is ex officio a member of the Advisory Board of each Panel)

NEW PROJECTS

Newsletter No. 6 (March 1961) listed the various projects being carried out by SMSG at that time. Since then, four new projects have been started.

SMALL PUBLICATIONS

In this project two kinds of publications will be prepared. The first will consist of supplementary material, especially for the more able students. The two small volumes "Essays in Number Theory" are typical examples.

The second kind of publication will consist of units, designed for classroom use, which can be used by teachers who would like to introduce some new ideas in their classes but are unable, for one reason or another, to undertake a new full year course. The SMSG "Junior High School Mathematics Units" are a typical example.

Presumably some material of this kind can be extracted from present SMSG publications, but the Panel (see p. 9) in charge of this project is free to search elsewhere for suitable material and also to arrange for the preparation of new material.

PROGRAMED LEARNING PROJECT

The School Mathematics Study Group has undertaken a project involving programed learning in mathematics, with special reference to the kind of mathematics illustrated in the texts already prepared by the SMSG. This project is being supervised by a Panel consisting of eight university and secondary school personnel in the fields of mathematics and psychology. (See list of Panel on Page 9.)

The Panel decided as a first step to have the mathematics in the SMSG *First Course in Algebra* programed. Two program forms were chosen: one consists of a series of constructed response questions with the answers provided; the other consists of passages followed by multiple choice questions with corrective material for incorrect alternatives selected.

In preparing these programs, the usual SMSG procedure will be followed; namely, writing by teams, reviewing, test-teaching, and revising. The program writing and reviewing is being done by a joint effort of mathematicians, teachers of mathematics, and psychologists. These materials will be tested in a variety of classrooms and instructional situations and revised on the basis of the experience gained. Such materials as prove effective will be made generally available.

TRANSLATION PROJECT

This project is devoted to translating into the Spanish language the sample texts prepared by SMSG for grades seven and nine. It is anticipated that there will also be a demand for grades eight and ten.

The primary purpose is to facilitate the mathematics education of the Spanish speaking citizens of the United States, principally in Puerto Rico. The translation into Spanish is essential for the removal of the language barrier to clarity of understanding.

Two centers are involved in the project. One is located at the University of Puerto Rico, the other at Stanford University. The texts will be available for use in September, 1962.

Preliminary chapters of the above material were presented at the Inter-American Conference on Mathematics Education held at Bogota, Colombia from December 4-9, 1961 where they aroused great interest.

The Advisory Panel for this project consists of Professors E. G. Begle, Stanford University, H. F. Fehr, Columbia University, Mariano Garcia, University of Puerto Rico, and Max Kramer, San Jose State College.

LONG TERM STUDY

The purpose of this study is to obtain information which can be used, on the one hand, by teachers, school administrators, parents, and other interested persons in evaluating the new mathematics curricula, and, on the other hand, by those who may wish to undertake further curriculum improvements in the future.

In this study students starting at three different grade levels will be followed for a period of five years. A large quantity of information on the students will be collected at the beginning, and the performance of the students will be measured from time to time, both in mathematics and also in science courses. More specifically, a group of students will be chosen who will enter fourth grade in September 1962 in schools where it is highly likely that their mathematics courses for the next five years will be of the kind prepared by SMSG. For reference purposes a similar group will be chosen in schools where it is highly likely that any of the mathematics courses for the next five years will be conventional. Similar pairs of groups of students will also be chosen at the 7th and 10th grade levels.

In order to factor out individual teacher differences, local school situations, etc., and also to take into account the expected drop-out due to moves from one school district to another, all of these groups must be large and will involve a substantial number of schools in various parts of the country.

A battery of tests will be administered at the beginning of the study which will measure aptitude and achievement in mathematics, attitudes, and primary mental abilities, and such other items as the Panel may decide on.

It is planned to measure achievement and attitudes of all students included in the study at the end of the third and fifth years. Selected samples will also be measured at the end of the first, second and fourth years. In addition, selected samples will be tested for performance in science

ORDER FORM

Quantity	*Title and Description (With Code Numbers)*	*Unit/Cost*	*Total*
	MATHEMATICS FOR THE ELEMENTARY SCHOOL		
	GRADE 4 Student text (E4-P) and Teacher Commentaries (CE4-1P) and CE4-2P)	3 vol. set $5.00	
	GRADE 5 Student text (E5-P) and Teacher Commentaries (CE5-1P and CE5-2P)	3 vol. set $5.00	
	GRADE 6 Student text (E6-P) and Teacher Commentaries (CE6-1P and CE6-2P)	3 vol. set $5.00	
	INTRODUCTION TO SECONDARY SCHOOL MATHEMATICS		
	Student texts (IS 1-4) and Teacher Commentaries (CIS 1-4)	8 vol. set $8.00	
	INTRODUCTION TO ALGEBRA		
	Student texts (IA 1-4) and Teacher Commentaries (CIA 1-4)	8 vol. set $8.00	
	GEOMETRY WITH COORDINATES		
	Student texts (GW 1-3) and Teachers Commentaries (CGW 1-3)	6 vol. set $6.00	
	STUDIES IN MATHEMATICS		
	Some Basic Mathematical Concepts (SM-1)	$ .80	
	Euclidean Geometry Based on Ruler and Protractor Axioms (SM-2)	$ .90	
	Structure of Elementary Algebra (SM-3)	$1.40	
	Geometry (SM-4)	$2.75	
	Concepts of Informal Geometry (SM-5)	$1.45	
	Number Systems (SM-6)	$2.40	
	Intuitive Geometry (SM-7)	$1.25	
	Concepts of Algebra (SM-8)	$2.40	
	JUNIOR HIGH SCHOOL MATHEMATICS UNITS		
	Volume 1. Number Systems, Student (U-1)	$ .70	
	Volume 1. Number Systems, Commentary (CU-1)	$ .70	
	Volume 2 Geometry, Student (U-2)	$ .60	
	Volume 2. Geometry, Commentary (CU-2)	$ .60	
	Volume 3. Applications, Student (U-3)	$ .40	
	Volume 3. Applications, Commentary (CU-3)	$ .40	
	PKG. Complete Sets (one each of the above 6 books)	$2.95	
	SUPPLEMENTARY UNITS		
	Mathematics for Junior High School Supplementary Units, Student (JSU)	$ .80	
	Mathematics for Junior High School Supplementary Units, Commentary (CJSU)	$ .65	

Quantity	*Title and Description (With Code Numbers)*	*Unit/Cost*	*Total*
	Essays on Number Theory I . . . . (HSU-1)	$ .30	
	Essays on Number Theory II . . . . (HSU-2)	$ .50	
	Development of the Real Number System . . . . (HSU-3)	$ .90	
	CONFERENCE REPORTS		
	Report of a Conference on Elementary School Mathematics . . . . (CR-1)	$ .40	
	Report of an Orientation Conference for SMSG Experimental Centers . (CR-2)	$1.00	
	Report of an Orientation Conference for SMSG Elementary School Experimental Centers . . . . (CR-3)	$1.00	

1. All SMSG publications are priced at cost.
2. Orders for less than $10.00 value accompanied by remittance, will be shipped at list prices postpaid.
3. Orders for $10.00 or more value from accredited schools will be allowed 10% discount but transportation will be added.
4. California schools and residents, please allow for 4% Sales Tax.

INVOICE ADDRESS

ORDER NO.____________

If this order is to be billed to a school or school district, please indicate full name and address.

SHIPPING ADDRESS
Use only if different than invoice address.

ORDERS BY INDIVIDUALS should be accompanied by full remittance. Please give full name and address.

SEND ORDERS AND MAKE CHECKS PAYABLE TO:

A. C. VROMAN, INC.
367 SOUTH PASADENA AVENUE
PASADENA, CALIFORNIA

All SMSG publications on this form are to be ordered from Vroman's.

A. C. Vroman, Inc., is a book depository firm and distributes these School Mathematics Study Group publications under a service contract.

courses each year. Students entering the 10th grade at the beginning of this study will be followed when they reach college as far as possible. Information on both achievement and career choices will be sought.

Both the initial battery of tests and the achievement tests to be used in this study will be planned and prepared under the supervision of the Panel on Tests, listed on page 9.

NEW PUBLICATIONS

TEXTBOOKS WITH TEACHER COMMENTARIES

Mathematics for the Elementary School

[Preliminary Edition]

Grade 4	3 vol.	$5.00
Grade 5	3 vol.	$5.00
Grade 6	3 vol.	$5.00

(For **Grade 4** the pupil text is in one volume and the teacher commentary in two volumes. This is also true for **Grade 5** and **Grade 6.**)

The SMSG materials for grades 4, 5, and 6 provide for review and extention of the operations of arithmetic which the pupils have had in each preceding grade. The study of the fundamental operations is extended in the familiar pattern to larger numbers, fractional numbers and decimals. The mathematics traditionally presented in these grades is included in **Mathematics for the Elementary School.**

In addition to this, the commutative, associative, and distributive properties are introduced early and used whenever appropriate. Numeration systems with different bases are used to clarify the meaning of place value, to deepen understanding of the operations, and to show common patterns in the different systems.

Addition and subtraction, as well as multiplication and division, are presented as inverse operations. The set concept is introduced early with considerable practice in the use of the language of sets. Facility in the use of both concept and language is strengthened in subsequent chapters which employ their use.

Careful attention is paid to use of language and symbols throughout the materials. When symbols facilitate the presentation they are introduced.

Adequate experience in problem solving is provided throughout the three grades to emphasize translation of relationships in problems into simple mathematical sentences and to establish firmly the skills required in the operations.

Approximately one-third of the materials in the three grades is geometry. Concepts from plane and solid geometry are presented together. The approach is intuitive and informal. Both material for the pupil and the suggestions for the teacher encourage the exploratory method of learning; the pupil is encouraged to draw conclusions from his study of many examples. No proofs are given or expected. Inferences expected of the pupil are left to be proved in the later work in formal geometry. From the use of a segment as a standard unit of length for measuring lengths, the transition to a standard unit of area to obtaining the measure of a plane region, and the use of a standard unit of volume to obtain the measure of the volume of a three-dimensional figure, proceeds naturally.

Arrangement of topics is influenced both by necessary sequence and attempt at variety. **Grade 4** is the beginning point and ideas introduced there are expanded in subsequent grades. If pupils are to begin with **Grade 5** or **Grade 6** they will need background in special topics provided in the preceding textbooks.

Introduction to Secondary School Mathematics

[Preliminary Edition]
Student texts (IS 1-4) and — 8 volume set
Teacher Commentaries (CIS 1-4) — Price $8.00

The **Introduction to Secondary School Mathematics** is an adjusted revision of *Mathematics for Junior High School* – Volumes I and II. The changes and adjustments which were made were prompted by a desire to simplify the presentation and reduce the reading difficulty. Explanatory sections were shortened and exercises added to lead the pupils through simple steps to appropriate conclusions. In a few cases concepts regarded as highly demanding were postponed but for the most part the initial S.M.S.G. 7th grade content was preserved. Abstract principles are carefully introduced by means of specific concrete examples.

The **Introduction to Secondary School Mathematics**—Volume I has 4 parts. The first 3 closely approximate *Mathematics for Junior High School* —Volume I. Part 4 is a selection of chapters from *Mathematics for Junior High School*—Volume II. The selection was made so that the emphasis is on preparation for algebra. Some topics in geometry usually associated with the 8th grade were necessarily and reluctantly omitted.

It is hoped that the **Introduction to Secondary School Mathematics** will serve to awaken the interest of a large group of junior high school pupils whose ability to learn mathematics has not been recognized and that it will build an understanding of fundamental concepts for pupils whose progress in mathematics has been blocked or hampered because of rote learning in an inappropriate curriculum. The title was selected with the express intent to permit the use of the material at the 7th or the 9th grade level.

Introduction to Algebra

[Preliminary Edition]

Student texts (IA 1-4) and	8 volume set
Teacher Commentaries (CIA 1-4)	Price $8.00

This SMSG course is designed for general mathematics students at about the ninth and tenth grade levels. It covers essentially the same material as the SMSG *First Course in Algebra* and it follows exactly the spirit of that course. (See SMSG *Newsletter* 6, page 22.) However, the language is simplified, the illustrations are extended, and the pace is slower, so that the course can be studied with profit over a period of 1½ to 2 years by some students who are not considered college capable. The range of ability best served by this course is being tested in an experimental program. Meanwhile, it is presumed that the range is about the middle 50 percent.

The course is based on structure properties of the real number system. Thus a student learns the nature of mathematics and strengthens his algebraic techniques by relating them to basic principles. The reading material and problems are designed to help the student discover ideas. The number line and simpler language of sets are useful in expressing these ideas. The material covered includes the usual topics of a first course in algebra.

Mathematics for High School Geometry with Coordinates

[Preliminary Edition] 6 volume
Student texts (GW 1-3) set
Teacher Commentaries (CGW 1-3) Price $6.00

This textbook is not a revision of the first SMSG geometry text, *Geometry* (revised edition) which was described in SMSG Newsletter No. 6, March 1961. However, they are similar in some respects:

1. The basic terminology and symbolism is the same.
2. Solid geometry is integrated with the plane, but long detailed deductive proofs in solid geometry have been eliminated.
3. Real numbers are used freely throughout the text, both in the theory and in the problems.
4. Accuracy in the statement and use of postulates, definitions and theorems is emphasized.
5. Geometric figures are considered as sets of points.
6. GW gives a synthetic treatment of the usual topics of congruence, similarity, parallelism, perpendicularity, area circles, spheres, etc.

Geometry With Coordinates does differ from the other SMSG geometry. It presents coordinate geometry as an integral part of the course as recommended by the Commission on Mathematics of the College Entrance Examination Board. Coordinates on a line are introduced in Chapter 3, coordinates in a plane in Chapter 8, and coordinates in space in Chapter 9. So coordinate as well as synthetic methods are used in proving theorems and solving original exercises starting with Chapter 3. Coordinate geometry in the plane is built in as an integral part of *GW*, but it was the last chapter in the other SMSG geometry. There is also a chapter on vectors. In the chapter on measurement of distance, emphasis is placed on the unit of measure.

GW is a deductive geometry using a complete set of postulates and definitions, but the set of postulates is not a minimal set. The basic postulates, definitions and theorems are motivated by having the students do certain experimental exercises and also by appealing to the students' intuition. It is hoped that this will make for better understanding of the postulates, definitions and theorems before the formal proofs of the theorems are given.

Briefly, **Geometry With Coordinates** is an integrated plane and solid geometry, including a chapter on vectors; with coordinate systems on a line, in a plane, and in space introduced as early as seems practical, so that both synthetic and coordinate methods may be used in the proofs of theorems and originals.

STUDIES IN MATHEMATICS

The next three volumes represent a continuation of the SMSG program of providing background material for teachers – in this case for teachers of the upper grades of the elementary school.

Such teachers are included in what is termed Level 1 in the "Recommendations of the Mathematical Association of America for the Training of Teachers of Mathematics."

These three volumes have been prepared in the belief that teachers at any level should not only thoroughly understand the mathematics they teach but should also have a good understanding of the basic concepts in the courses which their students will move on to. Accordingly, topics discussed in these three volumes are, for the most part, those normally met by students in grades 7 through 9.

VOLUME VI, NUMBER SYSTEMS

[Preliminary Edition] **Price $2.40**

A study of the structure of the number systems encountered in the elementary grades – the whole numbers, the non-negative rationals, and integers. A preliminary discussion of real numbers is also included.

Chapter Headings

1. What is Mathematics
2. Numeration
3. Whole Numbers
4. Rational Number System
5. Coordinates and Equations
6. Real Numbers

Appendix. Mathematical Systems

VOLUME VII, INTUITIVE GEOMETRY **Price $1.25**

[Preliminary Edition]

A companion volume to *Number Systems* intended to help elementary teachers develop sufficient subject matter competence in the mathematics of the elementary school program. Both the applications of number in geometry (measurement) and the relationships between geometric elements independent of number are presented to help form a foundation for the later study of geometry.

Chapter Headings

1. Non-Metric Geometry I
2. Measurement
3. Parallelograms and Triangles

4. Constructions and Congruent Triangles
5. Similar Triangles and Variation
6. Non-Metric Geometry II
7. Volumes and Surface Areas
8. Circles and Spheres
9. Relative Error

VOLUME VIII, CONCEPTS OF ALGEBRA

Price $2.40

[Preliminary Edition]

Concepts of Algebra provides an understanding of concepts that give elementary school mathematics the very necessary relationship to the total field of mathematics and particularly to subsequent mathematics in the secondary school.

In addition to assistance with terminology, the book has numerous exercises with a related answer section to enable the reader to test his understanding. These factors make it quite usable by individuals wishing to strengthen their background for elementary school mathematics. The following table of contents states the concepts emphasized.

Part

1. Numerals and Variables
2. Open Sentences and English Sentences
3. The Real Numbers
4. Properties of Order
5. Additive and Multiplicative Inverses

DISTRIBUTION OF SMSG PUBLICATIONS

There is only one source for each of the SMSG publications that may be ordered. The following three sections, I, II and III, relate the publications and the source from which they may be ordered. Care in directing orders to the correct location will avoid delay and disappointment.

I. Sample Textbooks for Grades 7-12

(Mathematics for Junior High School—Volumes I, and II; First Course in Algebra; Geometry; Intermediate Mathematics; Elementary Functions and Introduction to Matrix Algebra.) The Yale University Press will continue to publish these, and they will send an announcement concerning prices and improved distribution procedures for the next academic year to the entire mailing list for this Newsletter.

Yale University Press
92A Yale Station
New Haven, Connecticut

II. New Mathematical Library

Distribution of these Monographs to high schools has been taken over by the L. W. Singer Co. High school students and teachers may order these Monographs (Numbers: Rational and Irrational; What is Calculus About?; An Introduction to Inequalities; Geometric Inequalities; The Contest Problem Book; The Lore of Large Numbers), price 90¢ each, f.o.b. shipping point, from:

The L. W. Singer Co., Inc.
249 West Erie Boulevard
Syracuse 2, New York

The trade edition of these monographs will continue to be handled by Random House, Inc.

III. Other SMSG Publications

All other SMSG publications (Studies in Mathematics; Junior High School Mathematics Units; Supplementary Units; Conference Reports; and preliminary versions of sample textbooks) will be available **only** through the new SMSG distributor:

A. C. Vroman, Inc.
367 South Pasadena Avenue
Pasadena, California

These publications are listed in the Order Form included in this Newsletter. A. C. Vroman, Inc. is a school book depository.

If you are not now on our mailing list but wish to receive further issues of this NEWSLETTER, please request, by means of a post card, that your name be added to the mailing list.

SCHOOL
MATHEMATICS
STUDY GROUP

Newsletter No. 13

July 1962

SMSG PUBLICATIONS

CONTENTS

HOW THE TEXTS ARE PREPARED, 3

OTHER PUBLICATIONS, THEIR FUNCTION AND AVAILABILITY, 5

SMSG PUBLICATIONS LIST, 6

ORDER FORMS

"Preliminary Editions" only, Vroman, 9

"Revised Editions" only, Vroman, 10

"Sample Text Editions" only, Yale, 11

Other Publications, Vroman, 13

New Mathematical Library, Singer, 15

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SMSG

Since 1958 the School Mathematics Study Group has concerned itself with the improvement of teaching of mathematics in the schools of this country. The National Science Foundation has provided substantial funds in support of this endeavor. One of the prerequisites for the improvement of teaching of mathematics is an improved curriculum—one which takes account of the increasing use of mathematics and science in technology and in other areas of knowledge, and at the same time, reflects recent advances in mathematics itself.

One of the first projects undertaken by SMSG was to enlist a group of outstanding mathematicians and mathematics teachers to prepare a series of textbooks which would illustrate such an improved curriculum. It is not intended that these books be regarded as the only definitive way of presenting good mathematics to students at this level. Instead, they should be thought of as samples of the kind of improved curriculum that is needed and as sources of suggestions for authors of commercial textbooks.

It is the purpose of this Newsletter to make explicit the procedures used in generating and distributing SMSG materials, and to outline the method of ordering textbooks and other SMSG publications during the 1962-63 academic year.

HOW THE TEXTS ARE PREPARED

SMSG includes college and university mathematicians, teachers of mathematics at all levels, experts in education and psychology, and representatives of science and technology. Each summer since 1958 some of these individuals have gathered together on writing teams to produce text materials. In each case, the material generated by the writing team during the summer months was prepared for photo offset process, for use by schools during the following academic year, allowing the printer in some cases no more than a month to transform typed manuscript into paper bound books.

Each of the texts passes through three identifiable editions: *"Preliminary Edition"* *"Revised Edition"* *"Sample Text Edition"*. Other SMSG publications, their function and availability, are found listed on page 5 with the appropriate order forms at the end of this SMSG Newsletter. Since all SMSG publications are published at cost under support from The National Science Foundation, it should be borne in mind that NO FREE EXAMINATION COPIES can be sent to individuals or to organizations.

"Preliminary Edition" refers to experimental materials written by the summer writing teams for use in SMSG test-teaching centers throughout the country. The "Preliminary Edition" often consists of isolated units which are later to be incorporated into a single text. They tend to be somewhat experimental, attempting under controlled conditions to establish whether the content is appropriate for the grade for which it is written. At the moment, the *Programed First Course in Algebra* (Form MC) and (Form CR) and experimental materials for kindergarten through third grade are in this stage of development. These materials may be made available for inspection but only at some later time, to be announced in the SMSG Newsletter.

"Revised Edition" refers to the transition stage from the "Preliminary Edition" to the "Sample Text Edition". The "Preliminary Edition" having been test taught for one or more years, the "Revised Edition" reflects the revision completed during a summer writing session based on the information on teachability from classrooms using the "Preliminary Edition." Orders for the "Revised Edition" in classroom quantities for September, 1962 were required to be submitted by June to A. C. Vroman, Inc., distributor of "Revised Edition" materials. After classroom quantities have been filled, individual orders will be completed, probably around the first of January, 1963. *Geometry With Coordinates, Introduction to Secondary School Mathematics, Introduction to Algebra, and Mathematics for the Elementary School Grades 4 through 6* will be in this revised edition stage available only from A. C. Vroman, Inc. (see order form, page 10).

"Sample Text Edition" refers to the edition after the "Revised Edition," if no further revision by SMSG seems required. A non-profit press (Yale University Press at the moment) is contracted to publish the "Sample Texts"—"sample" in the sense that these are examples of mathematics which SMSG feels can be taught. When a sufficient number of commercially available textbooks appears on the market incorporating the suggestions and content of the School Mathematics Study Group materials, the SMSG "Sample Text Editions" will be allowed to go out of print. Notice of such a decision will always be announced well in advance of the date.

The current materials which are available in "Sample Text Edition" include The *Mathematics for Junior High School* series and *Mathematics for High School* series available from Yale University Press (see order form, page 11).

OTHER PUBLICATIONS, THEIR FUNCTION AND AVAILABILITY

In addition to "Sample Text" materials generated by the School Mathematics Study Group there are available several series of supplementary materials for students and teachers. One of these is SMSG *New Mathematical Library* for students. For teachers and teacher training institutions the SMSG has produced The SMSG *Studies in Mathematics*. In addition the SMSG has held a series of orientation conferences on the teaching of the new materials which have been written, the *Conference Reports* now being made generally available. Finally, a *Newsletter* announcing the progress of the projects, new publications, and activities of the School Mathematics Study Group is available at no cost by writing to:

SMSG – Cedar Hall
Stanford University
Stanford, California

SMSG PUBLICATIONS LIST

***P—Preliminary Edition (available for *individual* use until supply is exhausted)**

R—Revised Edition (available sometime after 1 January 1963)

Title and Description*	**Cost**	**Distributor**
MATHEMATICS FOR THE ELEMENTARY SCHOOL		
Grade 4, Text (E 4-P) and Commentary (CE 4-1P and CE 4-2P) 3 vol. set	$5.00	Vroman
SELECTED UNITS, GRADE 4 (E4150)	$ .75	Vroman
Grade 4, Text (E 4-1R) 3 parts	$2.00	Vroman
Grade 4, Commentary (CE 4-1R) 3 parts	$2.00	Vroman
Grade 5, Text (E 5-P) and Commentary (CE 5-1P and CE 5-2P) 3 vol. set	$5.00	Vroman
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Grade 5, Commentary (CE 5-1R) 3 parts	$2.00	Vroman
Grade 6, Text (E 6-P) and Commentary (CE 6-1P and CE 6-2P) 3 vol. set	$5.00	Vroman
Grade 6, Text (E 6-1R) 3 parts	$2.00	Vroman
Grade 6, Commentary (CE 6-1R) 3 parts	$2.00	Vroman
INTRODUCTION TO SECONDARY SCHOOL MATHEMATICS (Grade 7-8 modified) Text (IS 1-4P) and Commentaries (CIS 1-4P) 8 vol. set	$8.00	Vroman
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Volume I, Commentary (CISI-1R) 2 parts	$2.00	Vroman
Volume II, Text (ISII-1R) 2 parts	$2.00	Vroman
Volume II, Commentary (CISII-1R) 2 parts	$2.00	Vroman
MATHEMATICS FOR JUNIOR HIGH SCHOOL		
Volume I, Text (Grade 7) 2 parts	$3.00	Yale
Volume I, Commentary, 2 parts	$3.00	Yale
Volume II, Text (Grade 8) 2 parts	$3.00	Yale
Volume II, Commentary, 2 parts	$3.00	Yale
JUNIOR HIGH SCHOOL MATHEMATICS UNITS		
1. Number Systems, Text (U-1)	$ .70	Vroman
1. Number Systems, Commentary (CU-1)	$ .70	Vroman
2. Geometry, Text (U-2)	$ .60	Vroman
2. Geometry, Commentary (CU-2)	$ .60	Vroman

Title and Description*	**Cost**	**Distributor**
3. Applications, Text (U-3)	$ .40	Vroman
3. Applications, Commentary (CU-3)	$ .40	Vroman
PKG. Complete Set (one of each of 6 above)	$2.95	Vroman
SUPPLEMENTARY UNITS		
Jr. H.S. Suppl. Unit, Text (JSU)	$ .80	Vroman
Jr. H.S. Suppl. Unit, Commentary (CJSU)	$ .65	Vroman
Essays on Number Theory I (HSU-1)	$ .30	Vroman
Essays on Number Theory II (HSU-2)	$ .50	Vroman
Development of the Real Number System (HSU-3)	$ .90	Vroman
FIRST COURSE IN ALGEBRA		
Text, 2 parts	$3.00	Yale
FIRST COURSE IN ALGEBRA		
Commentary, 2 parts	$3.00	Yale
INTRODUCTION TO ALGEBRA		
Text (IA 1-4P) and Commentary (CIA 1-4P) 8 vol. set	$8.00	Vroman
INTRODUCTION TO ALGEBRA		
Text (IA-1R) 4 parts	$2.50	Vroman
Commentary (CIA-1R) 4 parts	$2.50	Vroman
GEOMETRY, Text, 2 parts	$3.00	Yale
GEOMETRY, Commentary, 2 parts	$3.00	Yale
GEOMETRY WITH COORDINATES		
Text (GW 1-3P) and Commentary (CGW 1-3P) 6 vol. set	$6.00	Vroman
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Commentary, 2 parts	$3.00	Yale
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ELEMENTARY FUNCTIONS, Commentary	$2.00	Yale
INTRODUCTION TO MATRIX ALGEBRA, Text	$2.00	Yale
INTRODUCTION TO MATRIX ALGEBRA, Commentary	$2.00	Yale

Title and Description*	Cost	Distributor
NEW MATHEMATICAL LIBRARY		
Numbers—Rational and Irrational by I. Niven	$ .90	Singer
What is Calculus About? by W. W. Sawyer	$ .90	Singer
An Introduction to Inequalities by E. Beckenbach and R. Bellman	$ .90	Singer
Geometric Inequalities by N. D. Kazarinoff	$ .90	Singer
The Contest Problem Book by C. T. Salkind	$ .90	Singer
The Lore of Large Numbers by P. J. Davis	$ .90	Singer
Uses of Infinity by L. Zippin	$ .90	Singer
Geometric Transformations by Yaglom-Shields	$ .90	Singer
STUDIES IN MATHEMATICS		
Euclidean Geometry Based on Ruler and Protractor Axioms (SM-2)	$ .90	Vroman
Structure of Elementary Algebra (SM-3)	$1.40	Vroman
Geometry (SM-4)	$2.75	Vroman
Concepts of Informal Geometry (SM-5)	$1.45	Vroman
Number Systems (SM-6)	$2.40	Vroman
Intuitive Geometry (SM-7)	$1.25	Vroman
Concepts of Algebra (SM-8)	$2.40	Vroman
CONFERENCE REPORTS		
Report of a Conference on Elementary School Mathematics (CR-1)	$ .40	Vroman
Report on an Orientation Conference for SMSG Experimental Centers (CR-2)	$1.00	Vroman
Report of an Orientation Conference for SMSG Elementary School Experimental Centers (CR-3)	$1.00	Vroman
Report for an Orientation Conference for SMSG Geometry With Coordinates (CR-4)	$ .40	Vroman

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	Grade 4, Text (E 4-P) and Commentary (CE 4-1P and CE 4-2P) 3 vol. set	$5.00	
	Grade 5, Text (E 5-P) and Commentary (CE 5-1P and CE 5-2P) 3 vol. set	$5.00	
	Grade 6, Text (E 6-P) and Commentary (CE 6-1P and CE 6-2P) 3 vol. set	$5.00	
	Selected Units, Grade 4, (E 4150)	$ .75	
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	INTRODUCTION TO ALGEBRA		
	Texts (IA 1-4P) and Commentaries (CIA 1-4P) 8 vol. set	$8.00	
	GEOMETRY WITH COORDINATES		
	Texts (GW 1-3P) and Commentaries (CGW 1-3P) 6 vol. set	$6.00	

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Orders for $10.00 or more value FROM ACCREDITED SCHOOLS will be allowed 10% discount but transportation will be added.

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	Volume II, Text (ISII-IR) 2 parts	$2.00	
	Volume II, Commentary (CISII-IR) 2 parts	$2.00	
INTRODUCTION TO ALGEBRA			
	Text (IA-IR) 4 parts	$2.50	
	Commentary (CIA-IR) 4 parts	$2.50	
GEOMETRY WITH COORDINATES			
	Text (GW-IR) 3 parts	$4.00	
	Commentary (CGW-IR) 3 parts	$4.00	

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92A Yale Station
New Haven, Conn.

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	Math for Junior High School Volume I, Commentary, 2 parts	$3.00	
	Math for Junior High School, Volume II, Text, 2 parts	$3.00	
	Math for Junior High School, Volume II, Commentary, 2 parts	$3.00	
	First Course in Algebra, Text, 2 parts	$3.00	
	First Course in Algebra, Commentary, 2 parts	$3.00	
	Geometry, Text, 2 parts	$3.00	
	Geometry, Commentary, 2 parts	$3.00	
	Intermediate Mathematics, Text, 2 parts	$3.00	
	Intermediate Mathematics, Commentary, 2 parts	$3.00	
	Elementary Functions, Text	$2.00	
	Elementary Functions, Commentary	$2.00	
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	Introduction to Matrix Algebra, Commentary	$2.00	

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See other side for
Information on Ordering Sample Text Editions.

INFORMATION ON ORDERING SAMPLE TEXT EDITIONS

All orders and/or correspondence should be addressed to:

YALE UNIVERSITY PRESS
School Mathematics Study Group Account
92A Yale Station
New Haven, Connecticut.

1. Texts and Commentaries must be listed separately on your order form.

2. Orders will **not** be acknowledged unless specifically requested and an acknowledgement form is provided.

3. An educational discount of 30% will be given to schools; a courtesy discount of 10% will be given to libraries, teachers, and other persons directly associated with the field of mathematics.

4. All regular shipping charges by book post or by truck will be paid by Yale University Press. If you request any special shipping services (airmail, special delivery, etc.) the extra charges will be billed on your invoice.

5. No free desk copies can be supplied.

6. Invoices will be sent when the books are shipped unless it is specifically requested that they be sent at a different time. Please read your invoice carefully. It will supply important information regarding method of shipment, date books were shipped, and the number of cartons in each shipment.

7. Place your order well in advance of your needs. Orders will be filled on a **first come, first served** basis.

8. Returns may be made without prior permission, subject to the following conditions: All returns must be sent back in salable condition, shipping charges prepaid. Our invoice number or numbers covering the purchase must accompany the shipment, preferably in an envelope attached to the outside of your parcel. (Your charge-back memo or other documents should be in the same envelope.)

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PUBLICATIONS

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367 South Pasadena Ave.
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	Structure of Elementary Algebra (SM-3)	$1.40	
	Geometry (SM-4)	$2.75	
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Newsletter No. 24

October 1966

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CONTENTS

AVAILABILITY OF SMSG TEXTS, 4

USE OF SMSG MATERIALS, 6

PRELIMINARY REPORT OF A NEW CURRICULUM PROJECT, 7

ZIP CODES, 12

REPORTS, 13–20

REPORT NO. 1
THE SMSG PROGRAMED LEARNING PROJECT, 13

ZIP CODE CARD, CENTERFOLD

REPORT NO. 2
THE SPECIAL CURRICULUM PROJECT, 20

NEW PUBLICATIONS, 28

ORDER FORM, 29

AVAILABILITY OF SMSG TEXTS

When the first SMSG texts appeared, two questions arose. The first was how long the texts should be kept available, and the second was what use the authors of commercial texts should be allowed to make of materials in the SMSG texts. Both these questions were discussed periodically in the SMSG Advisory Board, and a variety of possible policies were carefully examined.

Final resolution of both questions was reached during the past academic year. Reproduced below is the letter by means of which these decisions were communicated to the textbook publishing industry.

Dr. Austin J. McCaffrey
Executive Director
American Textbook Publishers Institute
432 Park Avenue South
New York, New York 10016

February 25, 1966

Dear Dr. McCaffrey:

The purpose of this letter is to transmit through you to the members of the American Textbook Publishers Institute current plans for the continued printing of the SMSG textbooks and our current policy on use of materials in these textbooks.

As you know, these textbooks were written because the mathematicians and classroom teachers who constitute SMSG felt that the mathematics textbooks available a decade ago were not presenting the kind of mathematics our students would need in the society they would find themselves in after finishing school. The group also felt that recommendations and exhortations would be far less effective than the existence of a set of sample textbooks illustrating our point of view and proven feasible in the classroom.

Once these textbooks were prepared, there proved to be a large demand for them. The group, therefore, decided to keep them available as long as they were needed, i.e., until the appearance through normal commercial channels of enough textbooks doing the same job.

In order to make sure that our texts were purchased solely because of the content and to avoid any appearance of competition with members of your Institute, we deliberately stopped short of a letterpress edition. All of our texts are produced by photo-offset from a typewritten manuscript. We have avoided the use of color. The texts are available only in paper covers and so have an estimated lifetime of less than two years and an estimated cost per student per year somewhat greater than that of the average hard cover text.

From time to time the SMSG Advisory Board has asked whether it is necessary to keep the SMSG texts in print. In order to assist the Advisory Board in its discussions, various kinds of information have been collected. Two years ago, for example, we sent questionnaires to a random sample of all schools that had purchased SMSG texts at any time since they first became available. Inspection of the returns showed, first, that a fairly substantial number of schools were switching to another text after two or three years use of an SMSG text and, second, in almost every such case the replacement text was one of the "modern" texts then becoming available. It may be that we are flattering ourselves, but we conclude that the SMSG texts, perhaps because of their extensive teachers' commentaries, provide an easy way for a school to move from a traditional program to a modern one.

We have also consulted extensively with state and city supervisors of mathematics. From them we learned that a non-negligible number of school systems have made heavy investments in inservice training of their teachers in order to use SMSG texts. These school systems have urgently requested that the SMSG texts be kept available long enough for them to get a reasonable return from this investment.

We also watch closely the year-by-year sales figures for each text. The general pattern has been an increase in sales each year until a text has been available for three years. At this point a steady decrease in yearly sales begins, and in fact, the decrease for the current school year was quite marked.

During each of the past two years we have reviewed very carefully a sample of the newly available commercial textbooks at the secondary school level. These reviews leave us with mixed feelings. On the one hand, we are extremely delighted to see how great the improvement in mathematics textbooks has been over the last decade. At every grade level there is now a variety of texts available with a sounder mathematical content and a more modern mathematical spirit than could be found in traditional texts. We only hope that it is proper for us to assume some of the credit for this improvement.

On the other hand, we were disappointed to see that most of these new texts have moved only part way from the traditional program toward what we consider feasible and appropriate for today's schools and students. We had hoped that by now many authors would have taken an SMSG text as a starting point and would have adapted, improved, and polished it into a form suitable for commercial publication.

The SMSG Advisory Board felt that this might be due to a too restrictive policy on use of SMSG materials or perhaps a misinterpretation of the policy as being too restrictive. Accordingly, this policy has now been revised as much as possible, consistent with our policy of insisting that everyone have access to these materials and that no one be given exclusive rights to them. Please note in particular that the possibility of a return of part of an author's royalties is no longer contemplated. A copy of this new policy is attached to this letter.

At the same time that this change in policy was decided, the SMSG Advisory Board came to a decision on the availability of SMSG texts. Since, on the one hand, an increasing number of improved textbooks are available and the sale of SMSG texts is decreasing but, on the other hand, the SMSG texts are still useful in helping teachers bridge the gap between a traditional program and a modern one, it has been decided that these texts will be kept in print as long as there is any demand for them and will be allowed to die a natural death when the demand disappears. The texts will be kept in their present format, and no revisions or editorial improvements will be made.

I trust that you will convey this information to the members of the American Textbook Publishers Institute and that at the same time you will convey to them our appreciation of the good work they are doing and our hope that they will continue their efforts.

Yours very truly,
E. G. Begle

EGB/san
Enclosure

Use of SMSG Materials

The primary purpose of SMSG is the improvement of mathematics in the schools. Therefore, the wide propagation of the ideas in SMSG materials, especially when thoughtfully reworked by competent authors, is clearly desirable. We encourage the free and widespread use of these ideas in the production of new textbooks. However, because the materials were produced with public funds, the use of SMSG materials should be supervised.

To implement these principles:

(1) SMSG continues to urge authors to improve, adapt, expand on and draw from materials prepared by SMSG.

(2) Permission to make verbatim use of current and projected SMSG materials must be secured from the Director of SMSG.

(3) Permission to use SMSG materials will be granted by the Director except in unusual circumstances.* Publications incorporating SMSG materials must include both an acknowledgment of the SMSG copyright (Yale University or Stanford University, as the case may be) and a disclaimer of SMSG endorsement. Exclusive license shall not be granted save in exceptional circumstances, and then only by specific action of the Advisory Board.

(4) In determining whether an author may use SMSG materials, no account will be taken of the author's connections with SMSG.

(5) In no case will permission to use SMSG materials be given until the SMSG publication has been generally available for two full years.

(6) In difficult cases, precedence should be given to the advantages of propagating the material.

* There are two exceptions. Permission to use extracts from a monograph in the New Mathematical Library must be secured from the author. At present, Modern Learning Aids has exclusive distribution rights for the filmed course for elementary school teachers.

PRELIMINARY REPORT ON A NEW CURRICULUM PROJECT

Since its beginning in 1958, SMSG has devoted the major part of its efforts to the preparation of sample textbooks. Of course many other activities were also undertaken – preparation of materials for teachers and for abler students, evaluations of various sorts, etc.– but all these together represent less time and manpower than the textbook effort.

This aspect of SMSG work is finished. With the revision of a computer text during the summer of 1966, a complete set of texts is now available covering the entire range from kindergarten to the end of high school.

Anticipating this, the SMSG Advisory Board has for the last few years been debating what further activities in curriculum development, if any, should be undertaken. A suggestion to revise and integrate the present series of texts, eliminating the redundancies and discontinuities resulting from the top-to-bottom order in which they were produced, was quickly rejected. The Board felt that such revisions and improvements could and should be undertaken by individual authors or teams of authors for commercial publication and that a single authoritative treatment by SMSG might well be stultifying.

However, the Board is convinced that curriculum development must continue indefinitely for a number of reasons. Most important of these is that changes in our society will continue to put new demands on the educational process and research and development will be needed to respond to these demands. For example, it has become clear only recently that statistics plays such an important role in our society that every student should be given a chance to understand the basic ideas of probability on which statistics is based. Similarly, the last decade has seen the use of electronic computers spread so rapidly and so widely that every student should be given the chance to learn enough of the relevant mathematics to be able to understand what computers can do and what they cannot do.

Another kind of change to which the school mathematics program should respond results from changes in mathematics curricula which have already been made. Changes in college mathematics courses seem to be resulting in greater mathematical understanding by those now entering the teaching profession. The widespread use of SMSG and similar texts during the past decade seems to have contributed similarly to an increased competence on the part of practicing teachers to handle new materials. Inservice training of teachers to prepare them for teaching SMSG has been a major factor in developing this increased competence. Changes in the elementary school curriculum seem to be providing students beginning the seventh grade with a background in mathematics different from what was the case ten years ago. Changes such as these may well make possible further changes in the secondary school program.

Most of the new high school texts, including those prepared by SMSG, still compartmentalize mathematics in the same way as was done in the past, e.g., ninth grade algebra, tenth grade geometry, etc. It is not unlikely that a different and more logical sequencing of topics might lead to a more efficient curriculum and one which would make possible the introduction of desirable new topics.

Curriculum research and development along the lines suggested by the above will undoubtedly require a large group effort and can best be carried out by an organization, such as SMSG, which can bring together the needed variety of teachers, mathematicians, educators, scientists, etc.

For all these reasons, the Advisory Board of SMSG has decided to take a second look at the mathematics curriculum for grades seven through twelve.

During the course of the discussion which led to this decision, two points were emphasized by the Board:

1. SMSG is not committing itself to the rapid production of a new series of texts for grades seven through twelve. Instead the emphasis in this new project will be on research and development. The question of implementation of the findings of the project will not be considered until later

2. A vigorous attack will be made on the problem of making clear to students the relevance of mathematics to problems of the real world. It is hoped that the curriculum to be developed will provide students with a clear understanding both of the nature of mathematical applications and also of the great variety of ways in which mathematics can be useful in our society.

Letters to SCIENCE and to the MATHEMATICS TEACHER early in 1966 announced this new project and urged all those interested in what mathematics is taught in our schools to submit suggestions and comments. Several hundred thoughtful letters were received and are being studied by those presently involved in the project.

The project is still in a preliminary, formative stage. A small steering committee met briefly last fall (1965) to provide some direction for a panel that met late in the winter (March, 1966) in New Orleans. The panel consisted of university mathematicians, applied mathematicians, and practicing secondary school mathematics teachers.

Two main guiding maxims emerged from their discussions:

1. The initial segment of the secondary school mathematics curriculum should be devoted to those mathematical concepts which all citizens should know in order to function satisfactorily in our rapidly expanding technological society. It was felt that capable students would be able to complete the study of this mathematics in three years or less, while the less able students might profitably spend four, five, or even six years completing the sequence.

2. The exposition of this mathematics for the average to slow-moving student will need to be satisfactorily developed if the project is to be a success.

With these maxims in mind the panel prepared in broad outline a list of topics which might constitute the mathematics program for grades seven, eight, and nine for capable students and also some suggestions of later topics.

It is hoped that many students finishing the proposed sequential seven-nine program in three years or less will continue in grades ten to twelve and will be able to study calculus as well as to undertake some work in linear algebra, vectors, probability, statistics, computers, matrices, and many other topics not normally found in today's seven-twelve program. It is quite clear that the proposed program will make it much easier for most students interested in careers in engineering and scientific fields to complete the present Advanced Placement Program in calculus and also to have some work in the other topics mentioned above.

The development of such a program for the last three years of high school poses some unique problems because of the diversity of the goals and interests of the group of students involved at this level. Some students will take only one more year of mathematics after the ninth grade, while some will want to be involved in studying mathematics for two or three more years. It appears that the group studying mathematics for three more years will essentially divide into two groups. One group would be those technically oriented students for whom the calculus will become the standard twelfth year course; the second group would consist of those equally capable students for whom additional work in probability and statistical inference, computers and computing, linear algebra, matrix algebra, etc., will possibly be more profitable at this time. The deliberations definitely indicated that more than one pathway will be available for capable students with different goals and interests.

During the summer (1966) about twenty practicing secondary school teachers and mathematicians met to begin to write detailed outlines of proposed materials for grades seven through nine. This outlining group adopted some broad general guidelines:

1. It is assumed that students entering the seventh grade will have had a modern program at least in the last few years of elementary school.

2. The mathematics to be introduced in the seventh, eighth, and ninth grades should not only be of value to all students as necessary for any intelligent, responsible future citizen regardless of occupation but should also provide a proper foundation for futher study of mathematics.

3. The work of other groups who have given considerable thought to the mathematics suitable for these grade levels should not be ignored. It does not seem reasonable to retrace the deliberation of the Cambridge Conference, the New Orleans planning group, etc.; instead we should respond responsibly to their recommendations.

4. New ideas and concepts are to be refined and generalized in a spiral manner over the three-year sequence.

5. Those aspects of current programs which have been successful in providing students with understanding and appreciation of the spirit and structure of mathematics should be retained, but at the same time the new program should deal realistically with meaningful applications of mathematics in many different fields.

6. Current mathematical developments that have arisen as major forces in the many new applications of mathematics, as well as in the development of mathematics itself, should be brought into the curriculum.

Some of the features of the seven-nine sequential program as it is now conceived by the outlining group are:

1. An attempt is made to fuse arithmetic, algebra, geometry in such a way that each one helps in supporting the development of the other whenever possible.

2. Geometry is presented in a concrete, intuitive, descriptive way. When a new concept is introduced, the focus is on its essential features.

Its more complex aspects and its relation to other concepts are treated later. The treatment of geometry fosters understanding of the concrete basis and the intuitive significance of geometric ideas before they are studied on a rather formal deductive basis.

3. One, two, and three dimensions will be considered in many geometrical questions where applicable. Coordinate geometry will appear as appropriate in helping describe sets of points algebraically. Solution sets of algebraic equations will be interpreted or described geometrically.

4. The process of model building in applied mathematics is the construction of a mathematical model that will help one to better understand a situation that occurs in a complex environment, since the analysis of such a mathematical situation often enables one to learn more about the original physical situation. This process will be developed at appropriate places in the seven-nine sequence.

5. Some relaxation in the present stress on structure may be noticeable, *but* structure is still definitely one of the unifying themes.

6. Short deductive sequences will appear in grades seven through nine using properties discovered in the intuitive development of both algebra and geometry. Some problems from number theory will be used to illustrate various methods of proof.

7. Appropriate materials will be included so that the student will have some appreciation for the mathematical activities associated with computer use and the role of computer in our technological society. Mathematical algorithms expressed in flow chart notation will offer an introduction to computer activity and require the student to have sufficient understanding of the mathematics for practical exploitation.

8. The concept of function will be considered early and will be used in many different types of mathematical content whenever appropriate.

9. The concept of a vector (displacement) appears in grades eight and nine.

10. Probability appears in grades seven and eight with some statistics in grade nine.

11. Numeration systems will get little treatment because it is assumed this topic will have been covered in elementary school.

12. The set concept and set notation will be used when convenient, but it will not be overplayed. It is assumed that the students have that concept from elementary school.

13. Notation and terminology introduced in

these grades will be compatible with present-day usage in mathematics.

During the course of the next few years the tentative outlines prepared this summer will be refined and extended to grades ten through twelve. Many of the units in these outlines will be tested experimentally in the classroom. Periodic progress reports of this project will appear in this newsletter.

ZIP CODES

Starting January 1, 1967, the U. S. post office will accept fourth class mail only if the zip code is included with each address. Since SMSG newsletters are mailed at the fourth class rate, it is necessary that the SMSG newsletter mailing list be updated to include zip codes.

If you wish to continue to receive SMSG newsletters, please fill out the address card in the centerfold of this newsletter and return it to SMSG headquarters.

Naturally, the above applies only to addresses within the USA. Those residing outside the USA will continue to receive SMSG newsletters as usual.

REPORTS

Reports on a number of SMSG projects, studies, and evaluations have appeared in earlier issues of this newsletter. Some studies, however, have resulted in reports too lengthy to be communicated in this fashion. Consequently, a new series of publications — SMSG Reports — has been started. The first two numbers in this series are now available. Single copies may be obtained by means of a post card request to

Reports
SMSG – Cedar Hall
Stanford University
Stanford, California 94305.

Summaries of these first two reports follow.

Report No. 1

THE SMSG PROGRAMED LEARNING PROJECT

In the late fifties, considerable interest in programed learning was generated from various sources, notably from a large number of writers and publishers. In response to the interest that was rampant at the time, the SMSG Ad Hoc Committee on Programed Learning was formed in 1961 to advise the Director of SMSG on procedures for a careful study of programed learning with specific reference to SMSG materials. The following recommendations were made at the initial meeting of the committee which took place at Harvard University on May 2, 1961:

1. Mathematicians from both high school and university levels should be taught to construct programs.

2. The Director of SMSG should locate a suitable number of qualified individuals who could devote part of their time during the 1961-62 academic year to the project.

3. Each section of the SMSG *First Course in Algebra* should be translated into programed form.

4. A variety of programs should be prepared.

5. The program writers should retain the content and sequence of the SMSG *First Course in Algebra* in order to make comparisons possible.

6. The problem of motivation should be kept in mind in writing the program.

7. Feedback should be obtained from students who try the program.

8. The objective of the first part of this SMSG

study should be to verify the hypothesis that the SMSG materials can be presented as effectively according to prescribed criteria, through programs as through conventional classroom-textbook procedures.

9. Evaluation of any program should involve the use of tests which require students to solve problems and recognize concepts outside the text.

Specific remarks spelled out the basic thinking behind these general recommendations. For example, the recommendation to translate the SMSG *First Course in Algebra* was motivated by the decision that programing this course would provide more useful information, even though it was felt that the SMSG 7th grade course would be technically easier to program. Again, for example, the variety of programs indicated was to include a constructed response program featuring small steps with some branching and a multiple choice program featuring a scrambled book format with explanations of incorrect answers. Occasional inclusion of larger steps in the constructed response program and variation in size of the unit in the multiple choice program were recommended for the purpose of experimentation.

With these recommendations of the preliminary committee, the Director appointed a Panel on Programed Learning consisting of R. C. Buck, University of Wisconsin; E. E. Hammond, Jr., Philips Andover Academy; L. D. Hawkinson, San Francisco Unified School District; J. G. Holland, Harvard University; W. J. McKeachie, University of Michigan; E. E. Moise, Harvard University; H. O. Pollak, Bell Telephone Laboratories; and D. W. Taylor, Yale University.

As a guide for the writing teams, a chapter by chapter summary and statement of objectives for the algebra course was prepared by V. H. Haag, H. O. Pollak, and C. E. Rickart who were members of the original writing team on the SMSG *First Course in Algebra*. Preliminary to the full scale production of materials, a *Manual for Programers* was drafted by Leander W. Smith, Coordinator of the Programed Learning Project. The manual was intended as a basic document to provide persons having a thorough knowledge of the mathematics with information that they would need for the actual construction of programs. It included some background material on the psychology, an overview of the state of the art of programing, specific recommendations for the analysis of the content of the SMSG *First Course in Algebra*, and notes on the construction of items.

The manual has since been revised and incorporated and published in a more comprehensive report of the SMSG Programed Learning Project.

Twelve writing centers were established to work on a part-time basis during the 1961-62 academic year. Each of the twelve writing centers consisted of three or four mathematicians and/or mathematics teachers whose acquaintance with SMSG *First Course in Algebra* stemmed from participation in the original writing team, the evaluation centers, or teaching SMSG mathematics to teachers. From each of these centers, representatives were assembled at Yale University for a workshop in August 1961, to acquaint a core of writers with programed instructional materials and the procedures for producing them; to prepare sample units in constructed response and multiple-choice formats which could serve as models for the center writing teams; and to take on writing assignments and suggestions for the organization of the twelve centers.

Participants in the Workshop were: D. W. Blakeslee, San Francisco; M. P. Bridgess, Boston; F. D. Jacobson, New Haven; H. Jones, Stillwater; W. W. Matson, Portland, Oregon; O. Peterson, Emporia, Kansas; P. O. Redgrave, Norwich, Connecticut; W. Storer, Des Moines; and H. Swain, Winnetka. The participants formed two writing teams; one to produce a constructed response program, the other to produce a multiple-choice program. With the experience of writing as a team in New Haven, the participants were then able to anticipate problems of producing programs and orienting other members of the centers.

Typical operational procedures for each of the writing centers consisted of first, internal criticism of a draft by members of the center and suggestions for refinement by the center chairman. With further refinement and editing of subsequent drafts, the revised drafts were sent to SMSG (then at Stanford University) for ditto reproduction and criticism by reviewers selected for this purpose. Each reviewer commented on the particular phase of the work in which he was best fitted: the psychologist commented on the programing per se, the mathematician on the correctness of the presentation of the mathematics, and the teachers on the appropriateness of language to the high school student. On receipt of the comments from the reviewers, the writing team reconsidered the language, mathematics, and programing technique, and prepared another draft for lithographing.

By spring (March 1962), the lithographed materials were made available to experimental centers. Prior to the use of these materials, a two part pre-test was administered to serve as an inventory of skills and concepts already in the command of the students. Writers were found to generate programed materials in the summer of 1962 for use in schools in the 1962-63 academic year. This writing group consisted of: John D. Baum, David W. Blakeslee, M. Philbrick Bridgess, Mrs. Marjorie French, James E. Gilbert, Arthur A. Hiatt, Stephen Hoffman, Mrs. Margaret Matchett, William W. Matson, Mrs. Persis O. Redgrave, Winfield Roberson, Robert E. Schweiker, William Storer, George Truscott, and Mrs. Helen Wehe. The participants were subdivided into two writing teams: one to revise the constructed response program, and the other to revise the multiple-choice program. Mr. Bridgess chaired the multiple-choice (Form MC) team, and Mr. Blakeslee chaired the constructed response (Form CR) team.

The writers of the multiple-choice program spent considerable time anticipating distractors (alternatives which require careful discrimination and knowledge of skills and/or concepts). The multiple-choice format enabled some preservation of text but suffered in its ability to elicit computational practice because of the form of presentation. The multiple-choice program that was produced that summer, although a scrambled book in form, was essentially a single-track program through which a student would proceed. Very little effort was made to capitalize on the multiple track program that might be possible given more time to write.

The writers of the constructed response program, on the other hand, found the fragmentation into small steps of the earlier version quite a problem. There was an initial tendency to use lengthy sets of problems for practice, over-cuing, and little synthetic material. Finding a need for synthesis, the writers redefined "frame." A "frame" had been conceived as a short passage, three to five lines, with a significant word or words missing. The response sought was then to contribute to the learning of the skill or concept. Each frame was then defined as a "bit" or "response eliciting item." As the summer progressed, the writers began to group exercises of similar nature into a box or frame; they then sought to use the "frame" as a means of identifying a recognizable step in the learning process. Each frame evolved as a conceptual unit; that is, an identifiable step toward the mastery of a skill or concept.

The ease of programing seemed to depend on the explicit structure of the content presented in the text and the independence of topics being developed. Programing of proofs was a challenge. To overcome part of the difficulty here, use was made of the constructed panel in the CR format. In this, the student was led to complete steps (or to supply reasons for steps) in a proof, transferring these responses to appropriate blanks in a Panel to complete the proof. The constructed proof was then compared with a model proof as it might appear. Thus, the Panel construction served to integrate the small pieces of the proof into a whole. Also, in the process of reconstruction, more than one pass were made in the proof.

In the spring of 1962, a design was planned for experimentation during the 1962-63 academic year. The design called for testing the following four modes of instruction:

MC – the multiple-choice programed text;
CR – the constructed response programed text;
MCR – the constructed response machine format (unbound CR in Koncept-O-Graph KOG-7);
9F – standard SMSG *First Course in Algebra.*

Each mode was to be used under certain of the following conditions:

Time-paced with a teacher:
  assigned with a due time or due date in blocks or sections with a teacher to control disbursement and to answer questions and discuss or explain any mathematics.
Time-paced with a monitor:
  assigned with a due time or due date in blocks or sections with a monitor to control disbursement and to answer nonmathematical questions.
Self-paced with a teacher:
  in classroom or home with a teacher to record time and to answer questions and discuss or explain any mathematics.
Self-paced with a monitor:
  in classroom or library with a monitor to record time and to answer nonmathematical questions.

Since teachers were often adverse to the notion of self-paced classes, almost all self-paced classes reverted to some form of time-pacing at some time during the year. Since these classes thus constituted a conglomeration of methods of presentation rather than pure form of self-pacing, they were eliminated from the analysis. The four treatment groups chosen for analysis were then, the MC (multiple-choice), CR (constructed response), MCR (constructed response, machine), and 9F (standard SMSG text) programs – all time-paced at 9th grade. The MCR classes were supervised by monitors while all others were supervised by teachers.

There was intense negative reaction from some of the teachers and students against the boredom they found resulting from the programs. Much of this reaction appeared to be generated by the sameness of format. In the case of the constructed response program, boredom was intensified among the faster students who were forced to go through long series of items designed primarily to teach the slower students. In the multiple-choice program, which provides for branching and skipping to care for the individual differences, there was objection to the "page-flipping" involved in the scrambled book technique; they also objected to being unable to review systematically without retracing all the steps in the program. In addition, the machines were found to be unsatisfactory. By January 1963, about half the machines were unusable because of mechanical deficiencies.

From the widespread dissatisfaction of teachers and students in the 1962-63 classes came the feeling that the "pure form" programs had made SMSG mathematics less palatable than the text on which they were based. Hence, in the spring of 1963 the Project Coordinator prepared a sample which combined the features of both earlier versions into a hybrid program. The features which were felt essential included:

(1) immediate confirmation on most constructed response items;

(2) confirmation and correction of errors on multiple-choice items without scrambling the text;

(3) inclusion of conventional textual material;

(4) inclusion of reviews and problem sets with confirmations given in the back of the book;

(5) inclusion of optional sections for more able students and for students who want or need additional help;

(6) inclusion of index, table of contents, tailor-made response sheets;

(7) variation in format to relieve boredom;

(8) provision for skipping items by those who have successfully completed criterion items.

The hybrid program, SMSG *Programed First Course in Algebra* (Form H), thus emerged in preliminary form after eight weeks of intensive writing in the summer of 1963. This writing team consisted of D. Blakeslee, M. P. Bridgess, F. Elder, J. C. Hammock, F. Jacobson, M. Matchett, W.

Matson, L. W. Smith, W. Storer, M. Wecheler, and M. Zelinka, chaired by Mrs. P. O. Redgrave. In 1963-64, an attempt was made to evaluate Form H in a large scale experiment with Forms CR, MC, 9F, and the SMSG *Introduction to Algebra* (IA), using common testing on all groups. (The SMSG *Introduction to Algebra* covers essentially the same material as the SMSG *First Course in Algebra,* but featuring slower pacing, more extensive illustrative materials, and simplified language.) For the evaluation, pre-tests and post-tests were administered. In the analysis, all items from the post-tests were coded as to the level of intellectual activity which they measured, Lo-cognitive measuring intellectual activity equal to or less than manipulating, and Hi-cognitive measuring intellectual activity greater than manipulating. A followup study was made in May 1964 of about 450 students who had been in the 1962-63 classes to determine whether retention of the algebra they had studied was related to the mode they had used.

During the 1964 SMSG Summer Writing Session, a writing team chaired by D. Blakeslee and consisting of W. G. Chinn, F. Jacobson, M. Matchett, W. Matson, and P. Redgrave comprehensively revised the hybrid text. This revision included significant changes in the mathematical content of 9F based on critical reviews by the PLP writers and analytical comments from members of the SMSG Advisory Board, notably those of P. D. Lax, New York University, and H. O. Pollack, Bell Telephone Laboratories. In addition, a Teacher Commentary was prepared. The final revision of the hybrid text concluded SMSG's examination of programed instruction.

In summary, the results of the 1962-63 study of the four modes indicated that both the constructed response (CR) and multiple-choice (MC) programs were significantly better than the standard SMSG text (9F) while there was no significant difference between CR and MC (all significance was at least at the .05 level). Also, both CR and MC were significantly better than the machine constructed response (MCR) while there was no significant difference between MCR and 9F. While many of the differences among treatment groups were significant, they were small in amount.

The results of the 1963-64 study indicated that with Lo-cognitive post-test measure of performance as the criterion variable, the MC, H, and 9F programs were better than CR, and both H and 9F were better than IA. While the difference between MC and IA was not significant, it approached significance. MC, H, and 9F tended to cluster together in their effectiveness to teach Lo-cognitive skills; likewise, CR and IA tended to be equally effective, though significantly less effective than the other three modes. With the Hi-cognitive post-test measure as the criterion variable, the results tended to indicate that all of the modes were equally effective for teaching Hi-cognitive skills. Only one difference was found to be significantly different: H > CR.

In the follow-up study of 1964, the null hypothesis of no difference among treatments was not disproved; that is, one year after completing the course, performance on the follow-up test was not significantly different for students who had used MC, CR, or 9F.

Original literature on programing did not provide for a program "mix." However, the mixed form tried in the 1964 writing had more clearly met the criteria of improved student performance and greater philosophical satisfaction to the writers. In view of this, an examination of programed materials that are feasible and promising should include such hybridization for consideration.

Report No. 2

THE SPECIAL CURRICULUM PROJECT:

Pilot Program on Mathematics Learning of Culturally Disadvantaged Primary School Children

Introduction

The purpose of the Special Curriculum Project is to study the readiness of culturally disadvantaged children for learning at school entrance and to follow their progress over the beginning school years with the aim of developing more effective materials for their learning of mathematics.

This project grew out of the Conference on Mathematics Education for Below Average Achievers sponsored by SMSG in April of 1964. On the basis of papers and discussions at this conference and of recommendations of an ad hoc committee, observations classes at the kindergarten and first grade levels were established for the 1964-65 school year in six cities

Fifteen classes, seven at kindergarten and eight at first grade, were selected as experimental classes. These classes were all located in disadvantaged areas of the six cities. All used the existing SMSG K and 1 texts, and, in addition, all were provided with a variety of manipulable materials for use in mathematics instruction.

Two kinds of comparison classes were included in the study. The first was designated as a socio-economic comparison group and included two kindergartens and two first grades in more advantaged areas of the same metropolitan areas wherein the experimental classes were located. The second comparison group included one kindergarten and one first grade class located in the same city as two of the experimental classes. These two classes used a curriculum other than SMSG.

The majority of these experimental and comparison groups were followed through the 1965-66 school year; however, the report here summarized deals only with individual testing of the children during the 1964-65 school year.

Design and Methodology

The study was based on the assumption that children from disadvantaged homes come to school with fewer experiences which enable academic success than do children from more advantaged homes. The emphasis of this pilot research was empirical, with major attention focused on measuring:

1) certain dimensions of readiness for school learning at kindergarten and first grade entrance;
2) certain facets of concept development;
3) development of particular mathematical concepts.

With the baseline of test scores at the beginning of the school year, it was then possible to measure change in performance over the school year with the combination of the SMSG elementary texts, special consultation assistance for the teachers, and the use of many concrete, manipulable materials to aid the children in understanding mathematical concepts.

The portion of the study here reported may be diagramed as follows:

School Year	**September**	**January**	**June**
Treatment	← SMSG Experimental Program → (Disadvantaged and Socio-Economic Comparison)		
	← Non-SMSG Curriculum → (Curriculum Comparison)		
Evaluation of Treatment Effects	Initial Inventory	Midyear Inventory	Final Inventory

The individually administered tests, labeled above as Initial, Midyear, and Final Inventory, were developed to minimize whatever differential might exist between the disadvantaged and more advantaged children in skills related to test-taking. The concepts and knowledge measured at each of the three individual testing sessions are shown in the following table.

INDIVIDUAL TESTS BY GRADE

Assessments Made	**Initial**		**Midyear**		**Final**	
	K	**1st**	**K**	**1st**	**K**	**1st**
Recognition						
Objects	X	X				
Photographs	X	X				
Drawings	X	X				
Vocabulary			X	X		
Visual Memory						
Objects	X	X			X	X
Pictures					X	X
Color Inventory						
Matching	X	X				
Naming	X	X			X	X
Identifying	X	X			X	X
Geometric Shapes						
Matching			X	X		
Naming			X	X	X	X
Identifying			X	X	X	X
Pairing				X		
Equivalent Sets			X	X		
Counting						
Buttons	X	X	X	X	X	
Members of a set			X	X	X	X
Rote	X	X			X	X
Rote by Tens						X
Number Symbols						
Identifying	X	X		X	X	X
Naming						X
Marking	X	X		X	X	X
Place Value						
Naming						X
Forming						X
Ordinal Number					X	X
Ordering			X	X	X	X
Classifying			X	X	X	X

As an explanation of the assessments listed in the above table, the Initial Inventory was planned as an evaluation of readiness of the children to learn mathematical concepts. This readiness is dependent upon experience and development in many areas. If the child is to learn to abstract and conceptualize from experience with concrete materials, it is important to ascertain whether he can recognize, by giving names to, the concrete materials being used (Recognition – Objects). Also, since color is an important classificatory principle in the early school years, it is important to learn whether the children can perceive and match the same hues, name the colors, and identify an object whose only property differentiating it from the others in the series is its color (Color Inventory).

Two other facets of cognitive development were assessed. The first was the ability of the child to make a transition from recognition of a concrete object to recognition of a photograph of it, and then to a line drawing of that object. The second was one kind of mediating response: visual memory. The progression from concrete to conceptual thought, upon which mathematical learning is based, requires that the child be able to form mental representations of objects he has previously seen but which are physically absent. Therefore, the visual memory assessment was used.

Assessment of performance on tasks more directly mathematical in nature included matching and naming of geometric shapes, counting, recognition of numerals, ordering objects by size, and classifying them by shape and color. A number of these assessments made in the initial testing were repeated or extended in the later testing sessions to ascertain growth through the school year. In addition, other kinds of tasks (Vocabulary, Pairing, Equivalent Sets, Place Value, Ordinal Number) were added.

Results and Discussion

The results are complex to present briefly since the study deals with experimental classes plus two kinds of comparison classes at two grade levels. In addition, the summary of results by curriculum and socio-economic level must be prefaced by the general finding that there was great variability within the classrooms of the disadvantaged children as well as great variability in performance between these classes at the end of the school year as well as at the beginning.

The results are presented by the specific assessments on the criterion of whether significant differences were found between samples. For this study the probability level accepted for significance of difference between groups was .01.

I. Initial Inventory

Significant differences which were found on the tests at the beginning of the school year were all in the direction of poorer performance of the disadvantaged children when compared with the more advantaged as well as the curriculum comparison group. The following table shows the specific assessments on which the experimental disadvantaged sample performed significantly less well.

Assessment	Grade Level	Comparison Group
Visual Memory — Objects	K	Curriculum
Color Matching	K	Curriculum
	1	Socio-economic
Color Naming	K, 1	Socio-economic
Color Identifying	K, 1	Socio-economic
Counting Objects (Buttons)	K, 1	Both
Rote Counting	1	Both
Recognition of Number Symbols	K	Socio-economic
	1	Both
Marking Number Symbols	1	Both

There were more specific tests at the beginning of the school year (Initial Inventory) on which no differences were found between the performance of the disadvantaged kindergarten sample and one or both of the two comparison groups than of the first grade sample. This supports findings from other studies which show an increasing lag in the performance of disadvantaged children as they progress from grade to grade.

The specific tests on which the disadvantaged kindergarten children showed no difference in performance when compared to one or both of the other two groups were:

Recognition – Objects, Photographs, Drawings
Visual Memory – Objects
Color – Matching, Naming, Identifying
Rote Counting
Number Symbol – Recognition, Marking

For the disadvantaged first grade children, the only tests on which no differences were obtained were:

Recognition – Photographs, Drawings
Color – Matching

II. Midyear Inventory

The tests at the middle of the school year measured primarily mathematical learning. At this point in time the pattern of differences between the groups was quite different from what was found at the beginning of the school year.

The experimental, disadvantaged kindergarten sample performed significantly better than did the curriculum comparison sample on Geometric Shapes – Matching and Naming, while the disadvantaged first grade children performed significantly better than the curriculum comparison children on Geometric Shapes – Naming, and on Pairing. These differences are likely attributable to differences in the curriculum but clearly attest to the ability of the disadvantaged children to learn the SMSG curriculum.

The performance of the disadvantaged first grade children was poorer than that of the socioeconomic comparison first graders on Vocabulary, Geometric Shapes – Naming, and Number Symbol Marking. They performed less well than did the curriculum comparison sample on number symbol recognition only.

There were no tests at midyear on which the disadvantaged kindergarten children performed at a significantly lower level than either of the two comparison kindergarten groups. This latter finding suggests that intervention at kindergarten with a structured mathematics program may diminish the discrepancy in mathematics performance between disadvantaged and more advantaged children which becomes readily apparent by third or fourth grade.

III. Final Inventory

By the end of the school year, the performance of the disadvantaged kindergarten children was statistically no different from either comparison group on most of the assessments made. Their performance on Visual Memory – Pictures was significantly poorer than that of the curriculum comparison kindergarten group. On Geometric Shapes – Naming and Identifying, they continued to perform better than the curriculum comparison group. On Counting Buttons and on Number Symbol Identification and Marking, the disadvantaged kindergarten children performed less well than the more advantaged kindergarteners, but these differences may be considered directional only since they reached the .02 and .05 levels of significance, respectively, instead of the accepted .01 level. By comparison to the significant differences between groups found at the beginning of the school year on these tests, their progress over the year was considerable.

For the first grade classes, the Final Inventory results were somewhat different. The one test on which the disadvantaged first graders performed significantly better than did the socio-economic comparison group was on Counting Members of a Set. On all other assessments in which significant differences were found, the performance of the disadvantaged first grade children was poorer than that of one or both of the comparison groups. Those assessments on which the performance of the experimental first graders was poorer than both comparison groups were:

Rote Counting
Rote Counting by Tens
Number Symbols – Naming and Marking
Place Value – Forming

On Ordinal Number, their performance was poorer than that of the curriculum comparison class.

It is important to emphasize, however, that the disadvantaged first graders' performance improved markedly over the course of the school year so that their performance on several tasks was not significantly different from the two comparison groups at the end of the school year although it had been at the beginning or middle of the year. This is the case for Number Symbols – Recognition, Geometric Shapes – Naming, Color – Naming and Identifying.

Summary and Implications

The report summarized has dealt with individual test results only, although classroom observations, teacher reports, and a group test administered at the end of the school year provided other kinds of data. Apart from the substantive findings reported, certain trends deserve mention. The variability in performance within the disadvantaged classes as well as between them was found with regularity although this intra- and inter-class variability could not be detailed in this summary. This variability demands a more careful look at the factors affecting the individual child's performance and suggests caution in grouping the findings of a number of classes on the basis that they are all composed of children who can be described as disadvantaged.

The changes in performance of the disadvantaged kindergarten children over the year were, on many of the tests, different from those changes observed in the first grade children. At this stage of the analysis, those changes which are effects of the differences in curriculum content at the two levels cannot be partialled out from those which may be attributed to the effects of earlier intervention with a structured mathematics program at the kindergarten grade.

Further analyses needed include intercorrelations among the various tests and between performance on the individual and group tests. Studies of the kinds of errors made, particularly on the number concept measures, may provide important insights into the process of learning mathematical ideas.

A further research need is for a longitudinal study to better evaluate the significance of providing experiences in mathematics at the kindergarten level on children's continued progress in the intermediate grades, a time at which the disadvantaged child's cumulative deficit has, in the past, become so apparent.

NEW PUBLICATIONS

SMSG has prepared units on probability for grades K through nine. These units are arranged in four volumes. For the primary grades the major emphasis is on the recognition that some events are uncertain. The notion of one event being "more likely" than another is developed. In the volume designed for the upper elementary grades, the appropriate mathematics of uncertainty is introduced.

A more formal treatment of probability for junior high school students is based on the language and symbolism of sets. Part I is largely intuitive and is designed for grade seven. Some of the more advanced material of Part II is best studied after some experience in algebra. The junior high school volumes are prepared in hybrid programed form and thus may be used as self-study units.

In all the units experiments and games are used to motivate mathematical generalizations. For the primary and intermediate grades, specially designed spinners have been prepared for this purpose and are available in classroom sets.

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SCHOOL
MATHEMATICS
STUDY GROUP

Newsletter No. 25

December 1966

ARTICULATION OF CONTENT OF SMSG TEXTS GRADES 7–10

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CONTENTS

Comments on Articulating SMSG *Mathematics for Junior High School* with students' prior SMSG experiences. p. 5

  Volume 1 Part I Chapters 1–8, 7
  Volume 1 Part II Chapters 9–14, 12
  Volume 2 Part I Chapters 1–6, 15
  Volume 2 Part II Chapters 7–13, 20

Comments on Articulating SMSG *First Course in Algebra* with students' prior SMSG experiences. p. 23

  Part I Chapters 1–9, 25
  Part II Chapters 10–17, 32

Comments on Articulating SMSG *Geometry* with students' prior SMSG experiences. p. 35

  Part I Chapters 1–10, 37
  Part II Chapters 11–17, 45

The textbooks which SMSG has prepared do not form a closely integrated sequence. In general, they are written for students whose previous training in mathematics was in a traditional program. Consequently, each text contains some material which has already been discussed in other SMSG texts for earlier grades.

A number of teachers have asked for advice as to how to proceed, especially in grades seven through ten, with students who have had previous SMSG experience.

A committee of classroom teachers who had had substantial experience with SMSG texts and with the problems of articulating them was asked to prepare suggestions and recommendations. This issue of the SMSG newsletter contains the report of this committee.

The members of the Articulation Committee were:

Sarah T. Herriot, Palo Alto, California (chairman)
Vincent Brant, Baltimore County, Maryland
Florence Elder, West Hempstead, New York
Tempie Franklin, Arlington, Virginia
Mildred Keiffer, Cincinnati, Ohio
Max Sobel, Montclair, New Jersey
Ruth Stone, Palo Alto, California

Editorial assistance in the preparation of the report was provided by:

Sarah Ferguson, San Francisco, California
Mary Huggins, Palo Alto, California

The purpose of the report of this SMSG committee is to help articulate the exploratory, informal work SMSG provides in the elementary years with the more unified and abstract content spiraling on through the junior and senior high school SMSG mathematics curriculum by offering suggestions for fitting prior SMSG experiences into a systematically enlarging body of knowledge.

It is the committee's consensus that omitting basically familiar material is rarely the answer to the articulation problem. Rather the committee sees as more productive the following alternatives:

1) identifying successively higher levels of presentation;
2) changing emphases to take full advantage of students' preparation; and
3) re-allocating class time to permit inclusion of applications such as those in SMSG *Mathematics Through Science* or in SMSG *Mathematics And Living Things.*

To assist in accomplishing these alternatives, this report lists references to topics from SMSG texts from grades four through eleven with each section of the SMSG texts for junior high school, algebra, and geometry, and distinguishes strictly new content from background material being unified. The teacher is strongly urged to review the sections cited, to draw examples from them for reteaching and/or supplementation, to verify that what may appear to be duplication of topics might be actually an ever widening spiral development of mathematical ideas.

The SMSG texts referred to are:

E-4: *Elementary School Mathematics,* Grade 4
E-5: *Elementary School Mathematics,* Grade 5
E-6: *Elementary School Mathematics,* Grade 6
J-1: *Mathematics for Junior High School* Volume 1, Parts I and II
J-2: *Mathematics for Junior High School* Volume 2, Parts I and II
9: *First Course in Algebra,* Parts I and II
10: *Geometry,* Parts I and II
11: *Intermediate Mathematics,* Parts I and II

References to SMSG texts are in the form (grade: chapter). (E-4:2), for instance, refers to the fourth grade text, Chapter 2.

COMMENTS ON ARTICULATING SMSG *MATHEMATICS FOR JUNIOR HIGH SCHOOL* WITH STUDENTS' PRIOR SMSG EXPERIENCES

For many years the junior high school years were the most barren ones in the mathematics curriculum. Students could expect no more than a rehash of arithmetic skills that they had learned in elementary grades. With the advent of the SMSG *Mathematics for Junior High School,* a refreshingly new approach was given that gave new impetus to the mathematics programs of these years. Subsequently, SMSG teams, well aware of the junior high context, wrote texts for fourth, fifth and sixth grades. This bulletin is written to help articulate the exploratory, informal work of the elementary years with the more unified and abstract content spiraling on through the junior and senior high school mathematics curriculum.

There is more than enough material in the two volumes of the SMSG *Mathematics for Junior High School* to provide for a rich two year experience. Although the learning process involves a certain amount of forgetting, and "refresher" activities must be provided even for students with previous SMSG backgrounds, care must be taken to avoid excessive repetition of basic concepts and skills. It is not necessary for pupils to repeat lessons they have mastered before. Time is needed for new ideas and new learning and every minute we can continue to save is already spoken for. On the other hand, it would be unwise to omit material entirely since backgrounds of some students will be uneven. Thus, some overlap is highly desirable. In fact, what may appear to be duplication of topics is actually an ever-widening, spiral development of concepts and ideas, broad generalizations, developing mathematical ideas in a more formal, mature manner than had been presented informally in earlier years.

In general, knowing that students come to them with previous SMSG preparation should cause teachers to provide a higher level of presentation, a changed emphasis, and a shortening of the time allotted to the various chapters of the text. This bulletin is written to aid teachers of junior high school mathematics in making judgments about the amount of overlap and pacing in developing the topics. Included in this section are references to topics which have been presented in previous

SMSG texts from grade 4 through grade 6, as well as to applications and extensions of junior high content to be found in high school algebra and geometry courses.

Suggested Time Schedule, Volume 1

Part I Chapter	Approximate Number of Days Recommended in Commentary	Time Recommended for Students With Several Years of SMSG Background	Total Number of Days
1	7	5	5
2	15	15	20
3	14	10	30
4	15	10	40
5	12	12–15	55
6	17	20	75
7	13	10	85
8	13	10	95
PART II			
9	18	20	115
10	12	10	125
11	13	15	140
12	11	10	150
13	10	10	160
14	5	5	165

Suggested Time Schedule, Volume 2

Although students with several years SMSG background have been introduced to many of the topics of Volume 2, the development in every chapter here is so much more sophisticated that the times recommended in the commentary should not be shortened. Needless to say, comments on articulation are designed to complement, not supplant, Teacher's Commentaries.

SMSG Mathematics for Junior High School, Volume 1, Part I

Chapter 1: (J-1:1)

WHAT IS MATHEMATICS? (pp. 1–21)
Strictly motivational.

Chapter 2: (J-1:2)

NUMERATION (pp. 21–61)

2-1 Cavemen's Numerals (pp. 21–26)
Motivational.

2-2 The Decimal System (pp. 26–30)
Rapid review of learnings from (E-4:2)*, (E-5:1) and (E-6:2).

2-3 Expanded Numerals and Exponential Notation (pp. 30–33)

Prior to directing class review of this topic, the teacher is well advised to become familiar with the extensive development of exponents in (E-6:1, 2, 10) in order to appreciate how compressed this section is. Work with exponents will continue in (J-2:3), where students study scientific notation. Supplementation now might well be drawn from sections on scientific notation in SMSG *Mathematics Through Science,* Part I, Chapter 2, and Part II, Appendix B.

2-4 Numerals in Base Seven (pp. 33–39),
2-5 Computation in Base Seven (pp. 39–48),
2-6 Changing From Base Ten to Base Seven (pp. 48–52),
2-7 Numerals in Other Bases (pp. 52–55), and
2-8 The Binary and Duodecimal Systems (pp. 56–61)

Numerals to bases other than ten were investigated in (E-4:2) and (E-5:1) without benefit of exponents. No work in bases other than ten appears at the sixth grade level. Consequently, the sections here combine at least two familiar topics (exponents and bases) in new ways.

Chapter 3: (J-1:3)

WHOLE NUMBERS (pp. 67–105)

3-1 Counting Numbers (pp. 67–70)

This section guides pupils to a new level of mathematical maturity because it abstracts the notion of one-to-one correspondence for the first time. The teacher might anticipate applying the idea in the geometric setting of (J-1:4).

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* References to SMSG texts are in the form (Grade: chapter). (E-4:2), for instance, refers to the fourth grade text, Chapter 2.

3-2 Commutative Properties for Whole Numbers (pp. 70–74),

3-3 Associative Properties for Whole Numbers (pp. 75–78), and

3-4 The Distributive Property (pp. 79–83)

In order to identify new abstractions to be taught in these sections, the teacher may retrace for himself explorations of properties in (E-4:3, 4, 7), (E-5:2, 3), and (E-6:2). It is precisely because of those prior explorations that the seventh grader is ready for symbolic statements of properties. Students may profitably augment the problem sets with exercises of their own creation.

3-5 Sets and the Closure Property (pp. 84–87)

The variety of sets familiar to SMSG students from (E-4:1, 5), (E-5:10), and (E-6:9) and early hints of closure in (E-4:3, p. 72; 7, p. 344) and (E-6:6, p. 371), should be known to the teacher before leading the class to the explicit idea of closure here for the first time.

3-6 Inverse Operations (pp. 88–92)

SMSG cultivates a non-verbal awareness of inverse operations in (E-4:3, 4), (E-5:3, 6), and (E-6:2). Before introducing it explicitly here, the teacher is urged to read those early sections as well as look ahead to (J-2:2), where inverse operations are first applied to equation solving.

3-7 Betweenness and The Number Line (p. 93)

The number line is introduced with counting numbers in (E-4:3, 10), with non-negative rationals in (E-5:6), with integers in (E-6:2, 4), and will be used for the reals in (J-2:6). In the latter section betweenness is basic to discussing irrational numbers.

3-8 The Number One (pp. 94–96) and

3-9 The Number Zero (pp. 97–101)

The teacher should be familiar with earlier intuitive development in (E-4:3, 4), (E-5:2), and (E-6:2) of the properties of one and zero in order to recognize the higher level of presentation here. Looking ahead, he can anticipate generalization in (J-2:6), where zero and one are seen as identity elements. Appropriate supplementation with student-devised exercises is indicated.

Chapter 4: (J-1:4)

NON-METRIC GEOMETRY (pp. 105–151)

Pupils who worked through SMSG *Elementary*

School Mathematics in grades four, five, and six had extensive experience with intuitive geometry. Essential vocabulary and notation will be familiar to them. Properties formally stated in this chapter have been explored on an informal basis. Sets and the symbols pupils use in writing about them were developed in the fourth grade and used in the fifth and sixth grades. Pupils have worked with simple closed curves and with plane regions. They should find much of this chapter a review on a higher level of material with which they have acquaintance but probably only spotty recall.

4-1 Points, Lines, and Space (pp. 106–108)

Pupils have used these words and ideas since the fourth grade. It should be sufficient to read and discuss the material for recall of ideas fundamental to Property 1. This is an opportunity for the teacher to listen while the pupils give evidence of what they know.

4-2 Planes (pp. 108–113)

Properties 2 and 3 formalize ideas suggested by experimentation with models in grade four. The teacher can rely on such remote experiences for little more than preparing pupils to accept the present level of formalism.

4-3 Names and Symbols (pp. 113–122)

This section builds on material explored in grade four. It may be read and discussed briefly, possibly with pupils demonstrating drawings and symbols at the board.

4-4 Intersection of Sets (pp. 122–125)

Symbols for union and intersection were used in (E-4:1, 5) and (E-6:9).

4-5 Intersections of Lines and Planes (pp. 125–130)

Much of this material was explored in (E-4:5); it is brought to a more abstract level here.

4-6 Segments (pp. 131–134)

Pupils have used the notation $\overline{AB}$ for segment AB since (E-4:9), so they should move quickly through these pages.

4-7 Separations (pp. 134–137)

Division of a plane by a line in the plane has been talked about since grade four but the word *separation* has not been specifically mentioned. This subtle notion is being cultivated patiently in anticipation of its significant application in geometry (10-3).

4-8 Angles and Triangles (pp. 138–141)

The ideas of this section were developed intuitively in (E-4:5, 8), where pupils met points interior to a region. The intersection of half planes, used to define the interior of an angle here and in (10-4), grows out of these earlier learnings.

4-9 One-to-One Correspondence (pp. 143–146)

This section is an expansion of (J-1:3, 4).

4-10 Simple Closed Curves (pp. 147–150)

Simple closed curves as such have not been discussed since their introduction in (E-4:5).

Chapter 5: (J-1:5)

FACTORING AND PRIMES (pp. 151–189)

5-1 Primes (pp. 151–155) and
5-2 Factors (pp. 155–160)

The teacher is urged to familiarize himself with investigations in (E-5:2) and (E-6:2, p. 53) to identify the new learnings emerging here as students use exponents in prime factorization for the first time.

5-3 Divisibility (pp. 160–164)

Aside from *Testing 2, 3, and 5 as Factors of a Number,* (E-5:2), the student has not worked with tests for divisibility.

5-4 Greatest Common Factor (pp. 164–169)

This section rapidly reviews ideas from (E-5:2, 6) and (E-6:2), before unifying them with recent work on exponents. Subsequent applications of the greatest common factor in simplifying nonnegative rational numbers (J-1:6), signed rationals (J-2:1), and rational expressions (9:9, 12) more than justify the spiraling of concepts seen here.

5-5 Remainders in Division (pp. 169–174)

Skills and understandings not refreshed in grade six after being developed in (E-4:7) and (E-5:2) are reviewed and advanced in this section.

5-6 Review (pp. 174–178)

Time saved earlier in the chapter may profitably be spent here.

5-7 Least Common Multiple (pp. 178–183)

The least common multiple was investigated at some length in (E-5:6) and very briefly mentioned in (E-6:2, 6). Here the skills and understandings must be reviewed thoroughly. The least common multiple of denominators will be required for non-negative rational numbers in (J-1:6), for signed rationals in (J-2:1), and for rational expressions in (9:9, 12).

Chapter 6: (J-1:6)

THE RATIONAL NUMBER SYSTEM (pp. 189–243)

Thorough re-teaching of all the sections in this chapter is recommended in view of the variety and complexity of the learnings centered about the rational number system. These topics are basic to study of percent (J-1:9), of signed rationals (J-2:1), and of rational expressions (9:9, 12).

Before undertaking such re-teaching via the advanced approach here, the teacher may become informed of the students' prior experiences by scanning the following content unified and extended in this chapter.

TOPIC	TEXT
Equivalent Fractions	(E-4:10), (E-5:6), (E-6:2,6)
Properties of Rational Numbers	(E-5:6), (E-6:2, 6)
Reciprocals	(E-6:2, 6)
Using the Number Line With Rational Numbers	(E-5:6), (E-6:2)
Multiplying Rational Numbers	(E-6:2)
Dividing Rational Numbers	(E-6:6)
Adding and Subtracting Rational Numbers	(E-5:6, 10), (E-6:2)
Ratios	(E-5:9)
Decimal Notation	(E-5:1, 6), (E-6:2, 6)
Order	(E-4:3, 10), (E-5:6)

Chapter 7: (J-1:7)

MEASUREMENT (pp. 243–298)

7-1 Counting and Measuring (pp. 243–249)

The idea of continuous quantities and the word *discrete* will be new. Properties of continuous quantities will also be new, but comparing areas and choosing units draw on preliminary work in (E-5:8).

7-2 Subdivision and Measurement (pp. 249–254) and

7-3 Subdividing Units of Measurement (pp. 254–260)

Subdivision is new but the vocabulary of measurement as on p. 251 appeared in both grades four and five.

7-4 Standard Units of Length (pp. 260–275)

Standard units of measure were explored in grades four and five, and now the division of inch and centimeter rulers are refined to 1/16 inch and 1 mm.

7-5 Precision of Measurement and the Greatest Possible Error (pp. 275–286)

These are strictly new ideas evolving from the preceding four sections. Greatest possible error, cast within the context of science, appears in SMSG *Mathematics and Living Things,* Part 1, Chapter 2.

7-6 Measurement of Angles (pp. 287–298)

Measurement of angles was carefully taught in Chapter 7 of the fifth grade text. In grade six, pupils constructed perpendicular lines using straightedge and compass. Here such earlier lessons are amplified.

Chapter 8: (J-1:8)

AREA, VOLUME, WEIGHT AND TIME (pp. 299–345)

8-1 Rectangle (pp. 299–316)

Some groundwork was laid with perimeters in (E-4:9) and with areas in (E-5:8).

8-2 Rectangular Prism (pp. 316–333)

Introductory work with solids, their faces, vertices, edges, and volumes in (E-4:8), (E-5:8), and (E-6:7) is augmented here in preparation for (J-1:10) and (J-2:11). Surface area and dimensionality are new.

8-3 Other Measures (pp. 333–344)

Computations with new units of measure.

SMSG Mathematics for Junior High School, Volume 1, Part II

Chapter 9: (J-1:9)

RATIOS, PERCENTS, AND DECIMALS (pp. 347–397)

This entire chapter is developmental, growing directly out of (J-1:6). Students with exclusively SMSG backgrounds meet percent for the first time here. Thorough teaching is demanded. Students should be encouraged to devise supplementary examples of operations with decimals, uses of equal ratios, and applications of percent.

Chapter 10: (J-1:10)

PARALLELS, PARALLELOGRAMS, TRIANGLES, AND RIGHT PRISMS
(pp. 397–463)

10-1 Two Lines in a Plane (pp. 397–404),

10-2 Three Lines in a Plane (pp. 404–409), and

10-3 Parallel Lines and Corresponding Angles (pp. 409–412)

The new ideas in these sections build on the work of (J-1:4, 8).

10-4 Converse (Turning a Statement Around) (pp. 412–416)

The teacher is referred to (E-4:6), where "if-then" thinking was used on a modest scale. The "if-then" form which was not employed explicitly again until this little section enhances the students' preparation for algebraic and geometric proofs in (9:7, 8) and (10:3).

10-5 Triangles (pp. 416–424) and

10-6 Angles of a Triangle (pp. 424–431)

These sections, just as the first three of the chapter, draw on (J-1:4, 8).

10-7 Parallelograms (pp. 431–438)

This is new content, relying upon section 3 above.

10-8 Areas of Parallelograms and Triangles (pp. 438–448)

The teacher is advised to scan in (E-5:8) the exploratory activities which represent the students' prior experience with areas of parallelograms and triangles. The teacher needs to be alert to the sophistication of the presentation of this section, for this material is extended in (J-2:11).

10-9 Right Prisms (pp. 448–462)

This section expands upon (J-1:8) and is preparatory to (J-2:11).

Chapter 11: (J-1:11)

CIRCLES (pp. 463–525)

11-1 Circles and the Compass (pp. 463–469)

Pupils are familiar with use of the compass and with such terms as *center* and *radius* from (E-4:5, 9) and (E-5:9). One lesson is sufficient for a rapid review.

11-2 Interiors and Intersections (pp. 469–475)

Intersections of circles with other circles, lines, and triangles in (E-6:9) and (J-1:4, 5) prepared students for definitions of interior and exterior of circles here. The term *concentric* is new.

11-3 Diameters and Tangents (pp. 475–481)

This new material is pointed toward (10:13).

11-4 Arcs (pp. 481–485)

This section synthesizes familiar ideas of separation (J-1:4) and arcs (E-5:4) and (E-6:9) to produce better understanding.

11-5 Central Angles (pp. 486–492)

In (E-6:9) pupils worked briefly with central angles.

11-6 Circumference of Circles (pp. 492–500)

This is new content, induced from Exercise 7 of the last section.

11-7 Area of a Circle (pp. 500–508)

The empirical approach to area through counting squares, familiar from (E-5:8) and (E-6:7), culminates in the formula for the area of a circle here. In (J-2:11) and (10:15), the area of a circle is seen as the limit of the areas of inscribed polygons.

11-8 Cylindrical Solids – Volume (pp. 509–514)

The teacher is urged to become familiar with the exploration of volumes in (E-6:7) to appreciate just how concise the present section is.

11-9 Cylindrical Solids – Surface Area (pp. 514–519)

Extends the thinking developed in (J-1:8).

11-10 Optional: Review of Chapters 10 and 11 (pp. 519–525)

If the teacher has succeeded in utilizing the students' SMSG background sufficiently, ample time will be available for these enrichment-type activities as well as some of the uses of circles and cylindrical solids found in SMSG *Mathematics and Living Things*, Part 2.

Chapter 12: (J-1:12)

MATHEMATICAL SYSTEMS (pp. 527–579)

Modular arithmetic here affords extension and reinforcement for some of the more abstract notions developed in earlier chapters of this text as background for algebra (9:3, 8).

Chapter 13: (J-1:13)

STATISTICS AND GRAPHS (pp. 579–609)

To appreciate fully the maturity of the materials in this chapter, the teacher is advised to inspect (E-6:8), the only prior SMSG content directly related to statistics. Here basic ideas and skills are refreshed and amplified. More extensive utilizations of graphing and statistical analysis of experimental data are available in SMSG *Mathematics and Living Things.*

Chapter 14: (J-1:14)

MATHEMATICS AT WORK IN SCIENCE (pp. 609–623)

Most of the students' mathematics up to this point have found applications internally – doing more mathematics. This chapter offers an external application which may very well lead to considering parts of Polya's *Mathematical Methods in Science*[1], and the SMSG volumes *Mathematics Through Science,* in anticipation of increasingly formal considerations of inverse variation in (J-2:9) and (9:14).

SMSG Mathematics for Junior High School, Volume 2, Part I

Chapter 1: (J-2:1)

RATIONAL NUMBERS AND COORDINATES (pp. 1–50)

1-1 The Number Line (pp. 1–7)

Uses of the number line to show addition and subtraction of whole numbers (E-4:3), of positive rationals (E-5:6), of integers (E-6:4), and to elucidate betweenness (J-1:3) are consolidated here in preparation for subsequent work in (J-2:6), (9:1), and (10:2).

1-2 Negative Rational Numbers (pp. 7–12)

SMSG students meet negative rationals here for the first time.

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[1] Polya, George. *Studies in Mathematics, Volume XI, Mathematical Methods in Science,* Stanford: School Mathematics Study Group, 1963.

1-3 Addition of Rational Numbers (pp. 12–21)

Opposites appeared in (E-6:4) with integers only; the *additive inverse* of a rational number is new at this point. Addition of signed rational numbers is new.

1-4 Coordinates (pp. 21–30)

An introduction to graphing on a line and in a plane was presented in (E-6:5), consequently only a brief review is needed here. (Exercises 1–4c serve as an adequate review.)

1-5 Graphs (pp. 30–34)

Graphing inequalities in a plane is new. The subject is expanded in both the ninth and tenth grade texts, consequently only an introduction is offered here.

1-6 Multiplication of Rational Numbers (pp. 34–41)

This is new material. The use of the distributive property as described on page 38 meshes with experiences in the elementary texts, e.g., (E-6:2), and in (J-1:6).

1-7 Division of Rational Numbers (pp. 42–46)

Reading the limited approach to this topic in (J-1:6) will aid the teacher in identifying new aspects of the topic presented here.

1-8 Subtraction of Rational Numbers (pp. 46–50)

Subtraction of integers was taught in (E-6:4); however, it probably needs to be re-taught in full detail here before moving on to subtraction of signed rational numbers.

Chapter 2: (J-2:2)

EQUATIONS (pp. 51–110)

2-1 Writing Number Phrases (pp. 51–57)

The concept of a number sentence is an integral part of the elementary SMSG texts, recurring in (E-4:3), (E-5:3), and (E-6:4). This section should require only brief treatment, possibly little more than a discussion of some of the exercises.

2-2 Writing Number Sentences (pp. 58–75)

This section provides a background for (9:3, 4) and should be taught as an introduction, not for complete mastery. Symbols expressing inequalities, employed only incidentally in the elementary texts, are essentially new.

2-3 Finding Solution Sets (pp. 75–90)

This is the student's first formal approach to finding the solution set of an open sentence. The teacher will benefit by reading (9:6) and (9:7) to see how the subject is taught the next year. Don't expect mastery here.

2-4 Solving Inequalities (pp. 91–93)

Use of the addition property of inequality, new to students here, will be studied again in (9:8) with the multiplication property of inequality. Treat this section as introductory.

2-5 Number Sentences With Two Unknowns (pp. 93–107)

The concepts here are new and difficult and will be studied in much greater detail in algebra. Even well-prepared students will be challenged. This is again introductory only.

Chapter 3: (J-2:3)

SCIENTIFIC NOTATION, DECIMALS, AND THE METRIC SYSTEM (pp. 111–155)

3-1 Large Numbers and Scientific Notation (pp. 111–117),
3-2 Calculating With Large Numbers (pp. 117–120),
3-3 Calculating With Small Numbers (pp. 120–125),
3-4 Multiplication: Large and Small Numbers (pp. 125–128), and
3-5 Division: Large and Small Numbers (pp. 128–133)

These sections provide the student's first encounters in regular SMSG texts with scientific notation. They jog his memory concerning positive exponents from (E-5:1), (E-6:1), and (J-1:2, 5), introduce negative exponents, and reinforce computational skills requisite for (9:10).

3-6 Use of Exponents in Multiplying and Dividing Decimals (pp. 133–137)

The treatments of decimals in (E-6:2) and (J-1:9) gave background for these new skills.

3-7 The Metric System: Metric Units of Length (pp. 137–145),
3-8 Metric Units of Area (pp. 145–148),
3-9 Metric Units of Volume (pp. 148–150), and
3-10 Metric Units of Mass and Capacity (pp. 150–155)

These last four sections pursue topics which spiral through (E-4:9), (E-5:8), (E-6:7), and (J-1:7).

Chapter 4: (J-2:4)

CONSTRUCTIONS, CONGRUENT TRIANGLES, AND THE PYTHAGOREAN PROPERTY (pp. 157–214)

4-1 Introduction to Mathematical Drawings and Constructions (pp. 157–162)

This section reviews and maintains the skill of using a protractor studied in (E-5:7) and (J-1:7).

4-2 Basic Constructions (pp. 162–171)

Straightedge and compass constructions were treated in (E-5:4) and (E-6:9). Review the present section very rapidly. (Exercises 4-2b alone may be adequate.) Constructions are again studied in (10:14).

4-3 Symmetry (pp. 172–176)

Line symmetry was discussed in (E-6:5). Only a brief review is needed, allowing time to elaborate on symmetry with respect to a point in Problem 7 of Exercises 4-3.

4-4 Congruent Triangles (pp. 176–183)

This section advances the student's thinking toward the goal of formal geometric proof at the tenth grade level. The teacher is urged to become familiar with investigation of conditions for congruence of triangles in (E-5:4) and of side and angle relationships of triangles in (E-6:3). Against this ample background, it should be possible to cover the material rapidly.

4-5 Showing Two Triangles to be Congruent (pp. 184–192)

Teach this section intuitively; few students are mature enough yet to benefit from the formal treatment given in the tenth grade. Constructing a perpendicular from a point to a line is refined here over the simple techniques of (E-6:9).

4-6 The Right Triangle (pp. 192–201)

Although right triangles were met in (E-4:8) and (E-6:3), this material is new. Develop it in detail. Because the treatment of square roots in (9:11) is succinct, give supplementary work with square root tables now.

4-7 One Proof of the Pythagorean Property (pp. 202–205)

This topic should be presented to every group with a background in SMSG elementary mathematics. A non-algebraic proof adaptable to overhead projectors appears in SMSG *Introduction to Secondary School Mathematics*, Volume 2, Part II, Chapter 19.

4-8 Quadrilaterals (pp. 206–208)

Consolidates and furthers studies from (E-4:5), (E-6:9), and (J-1:10).

4-9 Solids (pp. 208–214)

Sketching planes and three-dimensional figures is new. It will find its immediate application in (J-2:11) and thereafter in tenth grade geometry.

Chapter 5: (J-2:5)

RELATIVE ERROR (pp. 215–234)

New aspects of greatest possible error (J-1:7) and of scientific notation (J-2:3) are probed.

Chapter 6: (J-2:6)

REAL NUMBERS (pp. 235–279)

6-1 Review of Rational Numbers (pp. 235–240)

Material from (J-2:1) are reviewed here preparatory to investigating new properties of the rationals.

6-2 Density of Rational Numbers (pp. 240–245)

The teacher may want to read the treatment of this new concept in (9:1).

6-3 Decimal Representations for the Rational Numbers (pp. 245–250)

In (E-2:6) and (J-1:9) this topic was introduced briefly. The development here is in greater detail and contains many new ideas.

6-4 The Rational Number Corresponding to a Periodic Decimal (pp. 250–254)

This is new material. The students' strong background in factoring from (E-5:2), (E-6:2), and (J-1:5) should make explanation of terminating decimals easily understood.

6-5 Rational Points on the Number Line (pp. 255–257)

This new material is fundamental to the study of rational approximations of irrational numbers later in this chapter and in (9:11).

6-6 Irrational Numbers (pp. 257–262) and
6-7 A Decimal Representation for $\sqrt{2}$ (pp. 262–267)

New and difficult content. The term *irrational* is new; the number π was approximated in (J-1:11, pp. 496–498) via an argument similar to the one here for $\sqrt{2}$ without benefit of the vocabulary of irrational numbers. Guard against pressing for mastery of the proof; it recurs in (9:11). A somewhat simpler form appears in SMSG *Introduction to Secondary School Mathematics,* Volume 2, Part II, Chapter 20.

6-8 Irrational Numbers and the Real Number System (pp. 267–275) and
6-9 Geometric Properties of the Real Number Line (pp. 276–279)

This new and difficult content should be taught as introductory, not for mastery; it is simply background for intensive investigations appropriately reserved to (9:11) and (10:11).

SMSG Mathematics for Junior High School, Volume 2, Part II

Chapter 7: (J-2:7)
PERMUTATIONS AND SELECTIONS (pp. 281–310) and

Chapter 8: (J-2:8)
PROBABILITY (pp. 311–345)

The exploratory nature of activities in these chapters should not be obscured by occasional hints at formalization. Such formalization is relegated to the eleventh grade (11:14). Varied supplementary activities are provided in SMSG *Probability for Intermediate Grades* and SMSG *Introduction to Probability,* Parts 1 and 2.

Chapter 9: (J-2:9)
SIMILAR TRIANGLES AND VARIATION (pp. 347–416)

9-1 Indirect Measurement and Ratios (pp. 347–358) and
9-2 Trigonometric Ratios (pp. 358–365)

(E-5:9) and (J-1:6) constitute the students' prior experience with ratio; (E-6:6) and (J-2:1,2), their experience with coordinates in a plane. The unification and extensions here are pointed toward cultivating a feeling for trigonometric ratios, as opposed to formal manipulation which is appropriately postponed until (10: Appendix X) and (11:10).

9-3 Reading a Table (pp. 365–374)

Skills in reading tables are being expanded here. Interpolation is taught directly in (11:10) for the first time.

9-4 Slope of a Line (pp. 374–379)

This section grows out of the one immediately preceding it and provides some foundation for further study in (9:14). Slope is used as a tool in analyzing experimental data in SMSG *Mathematics Through Science,* Part I, Chapter 4.

9-5 Similar Triangles (pp. 379–384) and

9-6 Scale Drawings and Maps (pp. 384–391)

The new concept of similarity is founded in congruence, an idea familiar from (E-6:3), (J-1:7), and (J-2:4). These sections advance the students' facility with ratio by associating *proportional* and *scale* with ratios.

9-7 Kinds of Variation (pp. 392–395),

9-8 Direct Variation (pp. 396–400),

9-9 Inverse Variation (pp. 401–407), and

9-10 Other Types of Variation (Optional) (pp. 407–416)

Mathematical ideas continue to spiral here, uniting notions of slope from section 4 above, elaborating on *The Scientific Seesaw* from (J-1: 14), and extending understanding in the very kind of applications teachers of high school science expect their students to have in mathematics classes. SMSG *Mathematics Through Science,* Part II, Chapters 1, 2, and 3 contain abundant supplementary material.

Chapter 10: (J-2:10)

NON-METRIC GEOMETRY (pp. 417–449)

The teacher may profitably skim through nonmetric geometry in (J-1:4) and explorations with solids in (J-1:8, 10, 11) before moving into this developmental chapter. Looking ahead, SMSG *Geometry* integrates three-dimensional work with plane geometry on the supposition that the foundation from these investigations at the eighth grade level will survive a year's disuse while the student attends to algebra. Student-constructed models are indispensable to the present development.

Chapter 11: (J-2:11)

VOLUMES AND SURFACE AREAS (pp. 451–510)

Previous study of this topic occurred in (J-1:8, 10). Computational practice is more than an incidental bonus for working through this chapter ever on guard against pre-empting the formalism emphasized in tenth grade geometry (10:16).

Chapter 12: (J-2:12)

THE SPHERE (pp. 511–544)

The progression from the familiar to the new is rapid here; the exploratory emphasis must be preserved, with abundant use of concrete models. The benefits of the present investigations will be evident when spheres are formally treated in (10:16).

Chapter 13: (J-2:13)

WHAT NOBODY KNOWS ABOUT MATHEMATICS (pp. 545–579)

Time recommendations for the eighth grade presuppose reliance on the prior SMSG experiences cited throughout this report. It is consonant with the spirit of SMSG to resist any temptation to prolong practice beyond the recommendations and thereby to assure inclusion of this provocative final chapter.

COMMENTS ON ARTICULATING FOR SMSG *FIRST COURSE IN ALGEBRA* WITH STUDENTS' PRIOR SMSG EXPERIENCES

The greatest responsibility of a classroom teacher is probably that of making judgments. The teacher must make judgments about students, grades, selection of subject matter, and instructional procedures. Certainly, one of the most important phases in the entire evaluation process is the articulation of content as well as the level of presentation.

Articulation which involves the transition from one mathematics course to the next, has always presented a vexing problem to mathematics teachers. It is essential that teachers use diagnostic procedures in order to avoid duplication of content which students have met in previous SMSG texts. Furthermore, because the learning process involves a certain amount of forgetting, "refresher" activities must be provided even for students with previous SMSG backgrounds. It is not surprising that students become bored when supposedly new courses and topics in mathematics turn out to be dull rehash of topics which they have "discovered" and studied years earlier. On the other hand, it would be unwise to omit material entirely since backgrounds of some students will be uneven. Thus, some overlap is highly desirable. In fact, what may appear to be duplication of topics is actually an ever-widening spiral development of concepts and ideas. The Teacher's Commentary of the SMSG *First Course in Algebra* continually emphasizes the spiral approach.

At present, teachers are finding that students beginning the study of *First Course in Algebra* are better prepared than they were formerly because of previous SMSG background. In pacing and developing the course, teachers must take into account that some of the material in SMSG *First Course in Algebra* is not new.

Specifically, SMSG introduces sets, the number line, and some of the properties of $+$, $-$, $\times$, $\div$, as early as the fourth grade; factors and primes appear at the fifth grade; exponents and negative numbers at the sixth. The real number system is investigated at some depth at the eighth grade level.

In general, knowing that students come to them with previous SMSG preparation should cause teachers to provide a higher level of presentation, a changed emphasis, and a shortening of the time allotted to some of the early chapters of the text.

This section is written to aid teachers of *First Course in Algebra* in making judgments about the amount of overlap and pacing in developing the topics. Included in this bulletin are detailed references to topics which have been presented in SMSG texts for grades 4 through 8, and which will be presented in SMSG algebra and geometry texts.

Suggested Time Schedule

Part I Chapter	Approximate Number of Days Recommended in Commentary	Time Recommended for Students With Several Years of SMSG Background	Total Number of Days
1	6	1–2	2
2	8	4	6
3	18	10	16
4	10	3	19
5	10	5	24
6	10	2–3	27
7	—	6–10	37
8	—	6	43
9	—	12	55
PART II			
10	—	10	65
11	—	12–15	80
12	—	20–25	105
13	—	15–20	125
14	—	5	130
15	—	5–10	140
16	—	10–15	155
17	—	10	165

Rapidly covering the first five chapters, which draw heavily on pupils' prior SMSG experience, affords more than adequate time for new topics such as rational expressions, radicals, and completing the square.

SMSG First Course in Algebra, Part I

Chapter 1: (9:1)

SETS AND THE NUMBER LINE (pp. 1–18)

1-1 Sets and Subsets (pp. 1–6)

Such terms as *set, elements, union, intersection,* and *subsets,* as well as the symbols $\{\ \}$, ϕ, $\cup$, and $\cap$, appear as early as (E-4:1)* and (E-5:2, 6, 10). Set language is also used in (E-6:4, 5, 9), (J-1:3, 6), and (J-2:1, 4, 6) in developing the integers, rationals, and real numbers. Non-metric geometry (J-1:4) uses sets, while finite and infinite sets are identified in (J-1:12). The present section reviews only a few basic notions which, under the teacher's guidance, may prompt students to recall more details.

1-2 The Number Line (pp. 7–14)

The number line is used throughout SMSG texts: it is introduced in (E-4:3), and is used to interpret integers and rational numbers and their operations in (E-5:6), (E-6:4, 5), (J-1:3), and (J-2:1), and to work with real numbers in (J-2:6). This section, restricted as it is to the non-negative real numbers, may appear trivial unless judiciously supplemented, by student-constructed examples, for instance.

1-3 Addition and Multiplication on the Number Line (pp. 14–18)

The number line was used to describe operations on integers in (E-6:4) and rationals in (E-5:6), (E-6:2), and (J-2:1). Students with several years SMSG background require but a review here.

Chapter 2: (9:2)

NUMERALS AND VARIABLES (pp. 19–23)

2-1 Numerals and Numerical Phrases (pp. 19–23)

The distinction between *number* and *numeral* began in (E-4:2). (J-1:2) dealt with these terms in an explicit fashion. Numerical phrases and sentences appeared in (E-4:3), (E-5:9), and (J-2:2). Teachers should be able to go through this section as a rapid refresher exercise.

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* References to SMSG texts are in the form (Grade: chapter). (E-4:1), for instance, refers to the fourth grade text, Chapter 1.

2-2 Some Properties of Addition and Multiplication (pp. 23–28)

Binary operations defined on a set first appear in (J-1:12) through a discussion of abstract mathematical systems. Commutative and associative properties have been stated in a general fashion for addition and multiplication of counting numbers, whole numbers, integers, and rational numbers in (E-4:3, 4), (E-5:2, 6, 10), (E-6:2, 4, 6), (J-1:3, 6), and (J-2:1). *Mathematical Systems* (J-1:12) and *Real Numbers* (J-2:6) present these properties more formally. The review here concentrates on non-negative reals.

2-3 The Distributive Property (pp. 29–34)

The distributive property of multiplication over addition received considerable emphasis in (E-4:4), (E-5:3, 10), (E-6:2), (J-1:6), and (J-2:1) through numerical examples. It was handled more formally in (J-2:6). Its re-teaching here is basic to the concept of variable in the next section.

2-4 Variables (pp. 34–39)

The idea of a variable has been used informally since its introduction in (E-4:3), where, e.g., *n* represents the number which makes the sentence "$10 + n = 30$" true. This treatment is repeated in (E-4:4, 6, 7, 10), (E-5:3, 5, 9, 10), and (E-2:1, 2, 4, 5, 6, 7, 10). The seventh and eighth grade SMSG texts continue informal treatment of variables. The terms *variable* and *domain* will be new to students.

Chapter 3: (9:3)

SENTENCES AND PROPERTIES OF OPERATIONS (pp. 41–76)

3-1 Sentences, True and False (p. 41)

Mathematical sentences are introduced very early in SMSG texts and become an integral part of the texts. In (E-4:3) students are asked to judge sentences involving equations and inequalities as true or false, just as they are here in brief review exercises.

3-2 Open Sentences (pp. 42–44)

Open Sentences are treated informally in (E-4:3, 4, 6, 7, 10), (E-5:3, 6, 9, 10), (E-6:2, 4, 6, 10), (J-1:3, 6), and (J-2:2). Although the present review is cast within the context of the non-negative real numbers, the teacher may be well advised to exceed that limitation by admitting any negative real numbers students recall as they attack Problem Set 3-2c, for example.

3-3 Truth Sets of Open Sentences (pp. 45–47)

The phrase *truth set of an open sentence* will be new to students, although they have been finding truth sets informally since (E-4:3). On page 60 of (J-2:2) truth sets of open sentences involving equations and inequalities are treated explicitly but there they are called *solution sets*. Such prior experiences are extended in this section and should be recalled by specific examples.

3-4 Graphs of Truth Sets (pp. 47–48)

SMSG students explored graphing extensively in (J-2:1, 2); the present one-page review of the bare minimum is requisite to continued reinforcement and extension of concepts through the next six sections.

3-5 Sentences Involving Inequalities (p. 49)

Inequalities are developed gradually in (E-4:2, 10), (E-5:9), (E-6:4), (J-1:3), and (J-2:2); here exposition and exercises reinforce the basic vocabulary which may have been forgotten because of disuse.

3-6 Open Sentences Involving Inequalities (pp. 50–51)

Graphing inequalities is fascinating for many students; nonetheless, the skills required may be readily forgotten. The teacher, familiar with the development of graphing inequalities with rationals in (J-2:1, 2), can identify the few ideas being reviewed here and in section 3-8 below in anticipation of their extension to graphing inequalities with the reals in (9:5).

3-7 Sentences With More Than One Clause (pp. 52–53)

Compound sentences formed by using the connectives *and* and *or* are introduced very briefly in (J-2:2, pp. 68–70), along with a few graphs of the truth sets of easy compound sentences. The teacher can rely on little from that previous experience except that students may be vaguely aware of the topic, ready for thorough re-teaching. Encourage students to extend the problem sets to include examples of compound sentences using any real number.

3-8 Graphs of Truth Sets of Compound Open Sentences (pp. 54–55)

The subtle logic of these sentences warrants handling this section simply to reinforce all prior related topics.

3-9 Summary of Open Sentences (pp. 56–57)

The summary at this time provides a crystallization of ideas of the first eight sections and a springboard into the next sections on properties of addition and multiplication. Students with SMSG background should be successful in devising and graphing open sentences beyond those provided in the text.

3-10 Identity Elements (pp. 57–60)

Identity elements for addition and multiplication are introduced informally in (E-4:3, 5), (E-5:6), (E-6:2, 6), (J-1:3, 6), and (J-2:6) as the Property of Zero and the Property of One. Identity elements in an abstract mathematical system are discussed in (J-1:12). The quantifier *for every a* introduces new rigor in this section. Least common multiple is treated in (E-5:6) and (J-1:5, 6) but with simpler applications than developed here.

3-11 Closure (pp. 60–61)

Closure for whole numbers is presented informally in (E-4:3, 4). Discussion of this property for rational numbers appears in (E-5:6) and (E-6:2, 6). The concept of a closed set under an operation and the term *closure* appear in (J-1:3, 12). Closure for the reals is mentioned in (J-2:6) but without extensive rationalization and applications offered in sections 3-12 and 3-13 below; i.e., they are confined to non-negative real numbers.

3-12 Associative and Commutative Properties of Addition and Multiplication (pp. 61–66)

This section consolidates concepts previously investigated and reiterated in a variety of settings (see 2-2 and 3-11 above).

3-13 The Distributive Property (pp. 66–71)

The language of algebra is employed here for the first time to formalize and generalize much previous study (see 2-3 above) less abstractly derived from arithmetic. These topics have been so effectively spiraled that the student of SMSG will be convinced of the validity of the distributive property and ready to apply it.

3-14 Summary: Properties of Operations on Numbers of Arithmetic (pp. 71–76)

The summary of properties is for reference, *not* drill. A guide for teaching emphasis is implicit in the observation that less than one page is given to listing the properties; four pages are devoted to applications and extensions.

Chapter 4: (9:4)

OPEN SENTENCES AND ENGLISH SENTENCES (pp. 77–96)

Against the SMSG students' use of open phrases and sentences consistently since grade four, this section may cloy unless the teacher covers it at a fast pace and encourages pupils to utilize their background [illegible] posing comparable problems.

Chapter 5: (9:5)

THE REAL NUMBERS (pp. 96–120)

5-1 The Real Number Line (pp. 96–101)

The negative integers, opposites, and the numbering of the left half of the number line were taught initially in (E-6:4), negative rational numbers in (J-2:1), real numbers in (J-2:6). This section is essentially review and unification.

5-2 Order on the Real Number Line (pp. 102–107)

This is new content, building on many earlier presentations, particularly those of 3-5 above.

5-3 Opposites (pp. 108–112)

The idea of opposites was presented for integers in (E-6:4) and for rational numbers in (J-2:2). Relatively little difficulty should be encountered in a rapid review of the topic.

5-4 Absolute Value (pp. 113–120)

The concept of absolute value is new.

5-5 Summary (p. 118)

Numerical illustrations given by the student may reinforce the principles summarized from this chapter.

Chapter 6: (9:6)

PROPERTIES OF ADDITION (pp. 120–144)

6-1 Addition of Real Numbers (pp. 121–124),
6-2 Definition of Addition (pp. 124–129), and
6-3 Properties of Addition (pp. 129–132)

Students of SMSG had graphing and computational practice adding signed numbers in (E-6:4) and (J-2:1). Therefore, the initial sections of this chapter should be covered rapidly to restore skill while infusing the new interests of handling absolute values and looking at more formal statements of addition properties.

6-4 The Addition Property of Equality (pp. 132–135) and

6-5 The Additive Inverse (pp. 135–140)

The addition property of equality was introduced and applied to the solution of equations in (J-2:2). The concept of the inverse of an element under a binary operation was introduced in the discussion of abstract mathematical systems (J-1:12). Additive inverses for rational numbers were identified and applied to the solution of equations in (J-2:1). As a consequence of these exposures, SMSG students may solve many of the exercises in these sections by inspection and supplement them with examples of their own devising.

Chapter 7: (9:7)
PROPERTIES OF MULTIPLICATION (pp. 145–183)

7-1 Multiplication of Real Numbers (pp. 145–150),

7-2 Properties of Multiplication (pp. 150–156), and

7-3 Use of the Multiplication of Properties (pp. 156–159)

These sections draw together such previous topics for SMSG students as multiplication of signed numbers in (E-6:4) and (J-2:1), phrases and open sentences (9:2), and properties of multiplication (9:3). Although many of the exercises in these first three sections lend themselves to oral work and solution by inspection during rapid review, the content of the balance of Chapter 7 demands a slower pace.

7-4 Further Use of the Multiplication Properties (pp. 159–162)

Exponents on numerical bases are familiar to SMSG students from (E-6:1), (J-1:5), and (J-2:3). The teacher might use examples from these texts in introducing the new content of the present section.

7-5 Multiplicative Inverse (pp. 162–165),

7-6 Multiplication Property of Equality (pp. 165–166),

7-7 Solutions of Equations (pp. 167–172), and

7-8 Reciprocals (pp. 172–180)

Multiplicative inverses and reciprocals occur in (E-6:2), (J-1:6), and (J-2:1). The inverse of an element under a binary operation was discussed for abstract mathematical systems in (J-1:12). The multiplicative property of equality was applied to the solution of equations in (J-2:2). It is suggested that teachers rely on these preliminary experiences for little more than facilitating further presentations in the present sections.

7-9 The Two Basic Operations and the Inverse of a Number Under These Operations
(pp. 180–183)

This is a sophisticated summary of Chapters 6 and 7: in addition, multiplication, and their inverse operations.

Chapter 8: (9:8)

PROPERTIES OF ORDER (pp. 185–208)

Immediately following a brief review in section 8-1, the chapter emerges as strictly developmental, building new content directly on material from previous chapters of the First Course. Here again, encouraging students to handle as much of the practice material as they can orally not only capitalizes on their previous understanding but also alerts them to methods of attacking written problems demanding more structured approaches.

Chapter 9: (9:9)

SUBTRACTION AND DIVISION FOR REAL NUMBERS (pp. 209–245)

9-1 Definition of Subtraction (pp. 209–212) and

9-2 Properties of Subtraction (pp. 212–218)

The definition of subtraction for reals here is an extension of the definition of subtraction for rationals (J-2:1). Hence, the student with SMSG background will see these sections as aids to recalling and strengthening earlier understandings through rapid review leading into new, time-consuming, exercises.

9-3 Subtractions in Terms of Distance
(pp. 219–223)

This is new, drawing on the student's familiarity with the number line and on his recent experiences with absolute value (section 4 of chapter 5).

9-4 Division (pp. 223–229)

Division of rational numbers in (E-6:2), (J-1:6), and (J-2:1) precedes the extension in this section of division to the real numbers. Oral work on the simple practice materials will serve to renew skills prior to attempting the new and difficult exercises.

9-5 Common Names (pp. 229–233) and

9-6 Fractions (pp. 233–241)

The introductory remarks in these sections appeal to earlier understandings concerning rational numbers spiraling through (E-5:6), (E-6:2), (J-1:6), and (J-2:1). Students with SMSG backgrounds can be expected to proceed readily from the review-type numerical problems to the new algebraic rational expressions, where time saved earlier can productively be spent now.

SMSG First Course in Algebra, Part II

Chapter 10: (9:10)

FACTORS AND EXPONENTS (pp. 247–282)

10-1 Factors and Divisibility (pp. 247–251)

Factoring, with tests for factors, is introduced in (E-5:2). In this section, formal definitions of *factor* and *proper factor* are generalizations of definitions in (J-1:5). Review these numerical exercises rapidly.

10-2 Prime Numbers (pp. 252–254)

The terms *prime* and *composite* were encountered in (E-5:2) and (J-1:5); the Sieve of Eratosthenes, in (J-1:5). This numerical content is not time-consuming.

10-3 Prime Factorization (pp. 255–257)

The Fundamental Theorem of Arithmetic appeared as the Unique Factorization Property of Counting Numbers in (J-1:5), with the strictly numerical applications reviewed briefly here.

10-4 Adding and Subtracting Fractions (pp. 258–261)

The least common multiple and its application to adding fractions appeared in (E-5:6) and (J-1:5). Addition and subtraction of rational numbers spiraled upward in (J-1:8) and (J-2:1). In the present section, students progress from those earlier arithmetic manipulations to simple algebraic fractions.

10-5 Some Facts About Factors (pp. 261–266)

Although divisibility notions from (J-1:5) are extended and formalized here, the numerical emphasis requires little time.

10-6 Introduction to Exponents (pp. 266–269) and

10-7 Further Properties of Exponents (pp. 269–282)

These sections develop new content as they move rapidly from the numerical emphasis of section 7-4 above to algebraic expressions.

Chapter 11: (9:11)

RADICALS (pp. 283–311)

New content. Pre-algebra SMSG texts provide less reiteration of the mathematics of radicals than of many other topics; consequently, the teacher is advised to become familiar with the modest introductions to square root (J-2:4) and irrational numbers (J-2:6) before beginning this developmental chapter.

Chapter 12: (9:12)

POLYNOMIAL AND RATIONAL EXPRESSIONS (pp. 313–375)

The predominantly new content of this chapter reviews and significantly extends the mathematics of the preceding eleven chapters.

Chapter 13: (9:13)

TRUTH SETS OF OPEN SENTENCES (pp. 377–404)

The chapter synthesizes much fundamental mathematics from the students' background in SMSG. The idea of equivalent open sentences was introduced in (J-2:2), but there the term *equivalent equations* was employed. The addition property of equality appears in (J-2:2) and (9:6), the multiplication property of equality in (J-2:2) and (9:7), opposites of integers in (E-6:4) and (J-2:1), additive inverses for rational numbers in (J-2:1), multiplicative inverses in (J-2:6), reciprocals in (9:7). All of this background, and more, exists; nonetheless, some re-teaching is implicit.

Chapter 14: (9:14)

GRAPHS OF SENTENCES IN TWO VARIABLES (pp. 405–463)

14-1 The Real Number Plane (pp. 405–411), 14-2 Graphs of Open Sentences With Two Variables (pp. 411–422), and

14-3 Slopes and Intercepts (pp. 423–439)

(E-6:5) and (J-2:1, 2, 9) provided successive increments to skills and understandings being amplified in these sections.

14-4 Graphs of Open Sentences Involving Integers Only (pp. 440–447),

14-5 Graphs of Open Sentences Involving Absolute Value (pp. 448–463),

Chapter 15: (9:15)

SYSTEMS OF EQUATIONS AND INEQUALITIES (pp. 465–492),

Chapter 16: (9:16)

QUADRATIC POLYNOMIALS (pp. 493–510), and

Chapter 17: (9:17)

FUNCTIONS (pp. 511–545)

New concepts on these final hundred pages emerge directly from prior chapters. Every moment of time saved earlier in the *First Course* can be judiciously re-allocated to the sequence here culminating in the quadratic formula.

COMMENTS ON ARTICULATING SMSG *GEOMETRY* WITH STUDENTS' PRIOR SMSG EXPERIENCES

The students should be better prepared for the study of formal geometry in grade ten than they were formerly. They arrive not only with their customary first algebra of the year before, but also with years of informal geometry which has been interwoven into the curriculum of the elementary and the junior high school courses. Since the Birkhoff postulates are utilized in the *GEOMETRY*, the student's background, which includes real numbers, meshes well with the geometry, for the real numbers are used freely in measuring both distance and angles. Some of the *GEOMETRY* topics are essentially algebraic, giving the teacher many opportunities to reinforce and deepen the student's algebra instead of allowing his algebra to fade into disuse.

With the student's SMSG background the teacher must be aware not only of the amount of overlap but of the depth and length of exposure to topics. A spiral curriculum is considered essential but it must not circle in the same place, nor must it suddenly ascend too rapidly.

Omitting material is rarely the answer to the articulation problem, for too often the student has unexpected blank spots. The teacher's realization that the material, as such, is not new, should mean, however, (1) a higher level of presentation, (2) perhaps a changed emphasis, which takes advantage of the student's previous preparation, and, most often, (3) a shortening of allotted time. The *SMSG Geometry* which seemed impossibly long during the first year or so can now be completed with ease in a year or less by students who have studied the SMSG materials for several years previously.

It is deadly boring for students to "discover" and study topics presented at the same level in grade 10 that they "discovered" and studied two or three years earlier. Teachers who are unfamiliar with previous texts studied by their students can easily fall into this trap. In a spiral curriculum it is necessary to repeat materials, but this should be done with new insights and more depth. (It is impossible for textbook writers to consider all aspects of this problem, for some students will study the text with no previous intuitive geometry.) With the background of SMSG texts and with careful orientation by the teacher, the students are ready for a formal approach to geometry.

Suggested Time Schedule

Part I Chapter	Approximate Number of Days Recommended in Commentary	Time Recommended for Students With Several Years of SMSG Background	Total Number of Days
1	3	2	2
2	10	5–6	8
3	6	4	12
4	6	4	16
5	20	18	34
6	6	7	41
7	8	8	49
8	9	7	56
9	17	17	73
10	6	5	78
PART II			
11	10	10	88
12	15	16	104
13	13	14	118
14	10	10	128
15	5	8	136
16	8	8	144
17	20	20	164

Given the advantage of several years of SMSG background, students should move rapidly through the first four chapters of transition from their earlier intuitive geometry to the beginnings of formal proof. The teacher may wish to retrace for himself the explorations of informal geometry as cited from SMSG elementary and junior high school courses in order to identify new abstractions taught at the tenth grade level. It is precisely because of those prior investigations that students with SMSG preparations are ready for SMSG *Geometry*. Here again, comments on articulation are designed to complement, not supplant, the Teacher's Commentary.

SMSG Geometry, Part I

Chapter 1: (10:1)

COMMON SENSE AND ORGANIZED KNOWLEDGE (pp. 1–14)

1-1 Two Types of Problems (pp. 1-8)

This is an excellent introduction to the geometry if the teacher heeds the commentary and uses this section as it is intended and not primarily for algebra review for forgetful students. Articulation problems occur throughout the text not only in overlap of geometric ideas but in overcoming student weaknesses in algebra.

1-2 An Organized Logical Development of Geometry (pp. 8-14).

This section can be considered to be the introduction to the real meat of Chapter II. Even for those students with considerable background in intuitive geometry, this section needs imaginative expansion by the teacher, for it sets the stage for the chapters to follow. Students find it difficult to distinguish between intuitive geometry and the very beginning of formal geometry. The logical development of the course cannot be overstressed at this point and in the next few chapters. It is vital that students accept the objectives of the course if they are to enjoy geometry to the fullest.

The spirit of earlier exposure to geometry in SMSG texts has been to have the student discover geometric principles and learn correct terminology and definitions. Little has been said about formal proofs in any texts before the first year algebra. The algebraic proofs, while fairly rigorous for the level, are difficult for most students and are not sufficient in themselves to show students the importance and necessity of proof in the study of a logical development of Euclidean geometry.

Chapter 2: (10:2)

SETS, REAL NUMBER AND LINES (pp. 15–51)

2-1 Sets (pp. 15–21),

2-2 The Real Numbers (pp. 21–27), and

2-3 The Absolute Value (pp. 27–30)

Well-prepared students can study these sections in a day or a day and one-half. The ideas and vocabulary of sets are quite familiar to students with SMSG experience from (E-4:1, 5)*, (J-1:3, 4), and (9:1). Real numbers are introduced in full in (J-2:1, 6) and (9:1, 5), rationals appear in (E-4:10) and (J-1:6), the integers in (E-6:4). Order relationships and inequalities recur in (E-4:2), (E-4:10), (E-5:9), (E-6:4), (J-1:3), and (J-2:1); a formal treatment of their properties occurs in (9:8). Absolute value is defined in (9:5).

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* References to SMSG texts are in the form (Grade: Chapter). (E-4:1, 5) for instance, refers to the fourth grade text, Chapters 1 and 5.

2-4 Measurement of Distance (pp. 30–33)

The distance between points as the measure of a segment is introduced without the concept of absolute value in (E-4:9), (E-6:5), and (J-1:7), and with it in (9:9). Units of measure of distance were studied in (E-4:9) and (J-1:7). The teacher is urged to become familiar with those earlier approaches in order to clarify the new emphasis that the number we get as a measure of distance depends upon the unit of measure.

2-5 A Choice of a Unit of Distance (pp. 33–34) and

2-6 An Infinite Ruler (pp. 35–39)

These sections build postulates on such essentially review materials as coordinate systems from (E-6:5), (J-2:1), and (9:14).

2-7 The Ruler Placement Postulates. Betweenness, Segments and Rays (pp. 39–48)

Even though most of the ideas will not be strange to the class, the emphasis given to this familiar material by the teacher at the tenth grade level must be more sophisticated than that of the junior high school teacher of intuitive geometry.

Lines, rays and segments appear in (E-4:5) and frequently thereafter. The idea of betweenness is developed in (J-1:4). Under the heading of *property* or *law* several of the postulates have been stated. Under the heading of *postulates,* several theorems have been stated. These provide excellent opportunities for the teachers to discuss why we develop mathematics differently at various levels, emphasizing the maturity of the student as an important factor.

Contrary to the convention reiterated as recently as page 23, number lines on pages 39 and 40 have smaller numbers to the right of larger ones. With strong students, the teacher might well refer back to the rigorous test for $y > x$, ($y > x$ if $y - x > 0$), suggested on page 219, SMSG *First Course in Algebra*.

Students should be provided with problems where the necessity of an absolute value sign in

3) discussing such questions as:
   a) Can two lines separate a plane into three regions? (Problem Set 3-3, Problem 1[i];
   b) Can three lines in a plane ever separate the plane into four regions? (Problem Set 3-8, Problem 14);
   c) Problem 2(f) of Set 3-2, "Two parallel lines determine a plane."

These problems can contribute to the student's understanding if handled informally as intended.

3-2 Theorems in the Form of Hypothesis and Conclusion (pp. 60–62)

This section is important to the remainder of the course and could use expansion, not contraction. The student's prior exposures are from (J-1:10), where "if-then" statements and their converses are mentioned, and (9:7) where "if and only if" statements are discussed. Supplementation with negation and the contrapositive would point out that implication is just one of many forms of statements used in logic.

3-3 Convex Sets (pp. 62–68)

This is new content, founded on work with regions and separations in (J-1:4) and (J-2:11).

Chapter 4: (10:4)

ANGLES AND TRIANGLES (pp. 71–95)

4-1 The Basic Definitions (pp. 71–77)

Students with SMSG background have worked intermittently with angles and triangles in (E-4:5) and (J-1:4, 10). Knowledge of words such as *side, vertex, interior, exterior,* and notations such as $\angle$ BAC and ΔBAC are assumed in subsequent material. Though most of the ideas are familiar, omission is not the solution. The students need review given in a brief class discussion.

4-2 Remarks on Angles (pp. 77–78)

A few brief remarks by the teacher will suffice; students will not draw on this content until they study trigonometry.

4-3 Measurement of Angles (pp. 79–85)

Use of the protractor occurs in (E-5:7) and (J-1:7); however, since most protractors have two scales instead of the single one printed on pages 79–83, a review is essential.

4-4 Perpendicularity, Right Angles and Congruences of Angles (pp. 85–92)

Categories of angles appear in seventh grade: right, acute and obtuse in (J-1:7); supplementary and vertical in (J-1:10); Theorem 4-7 is stated in (J-1:10). Perpendicularity of lines, segments, and rays is presented in (J-1:7) and (J-2:4). Congruence of angles is defined in (E-4:8) and (J-1:7). Work with triangles appeared in (E-5:4) and (E-6:3). Deductive reasoning in "if-then" form is mentioned in (J-1:1, 10) and in (9:7). Some re-teaching is required to advance undertanding.

Review Problems (pp. 92–95)

These exercises provide an opportunity to unify all the ideas of this chapter before launching into congruences. However, the teacher must be alert not to bore students with too much repetition and yet not to take too much for granted in their understanding before their experience of writing proofs. An expert teacher with probing questions can distinguish between glibness, indicating familiarity with terms but only vague comprehension of the meaning, and real understanding though not always well-verbalized. Again, the emphasis must be on the steady build-up of definitions, postulates, and theorems.

Chapter 5: (10:5)

CONGRUENCES (pp. 97–154)

5-1 The Idea of a Congruence (pp. 97–108)

Congruence of triangles is investigated in (E-4:8), (E-5:4), (J-1:7), and (J-2:4). The technique used in each is one of exploration. The idea of correspondence is mentioned in (J-1:4). This section should be treated as a review, except for the notation used in writing correspondences between triangles.

5-2 Congruence Between Triangles (pp. 109–115) and

5-3 The Basic Congruence Postulate (pp. 115–117)

The properties of congruent triangles are developed in the fifth grade through tracing geometric figures and placing them on others. Conditions for congruence, S.S.S., S.A.S., and A.S.A., are postulated in (J-2:4). Treat the present sections as review.

5-4 Writing Your Own Proofs (pp. 117–122)

Treat this section as new work; proofs in previous work in geometry have been informal.

5-5 Overlapping Triangles. Using the Figure in Statements (pp. 123–126)

Treat as new work.

5-6 The Isosceles Triangle Theorem. The Angle Bisector Theorem (pp. 127–132)

Properties of an isosceles triangle are explored in the fourth, fifth, and sixth grades. Theorem 5-2 is stated in (E-6:3) and (J-1:10); however, no proof is given. Definitions of scalene, isosceles, and equiangular triangles are covered in (J-1:10). The bisector of an angle is defined in the sixth grade. Theorem 5-3 is new here. In (J-2:4) the student constructs the median of a triangle but it is not named as such.

5-7 The Angle Side Angle Theorem (pp. 132–136)

The A.S.A. theorem was postulated in the eighth grade. This is the first time a formal proof has been given.

5-8 The Side Side Side Theorem (pp. 137–147)

This theorem was postulated in (J-2:4). The exercises are more difficult than earlier ones and the fallacy of a circular argument is new. Remind the students of the fallacious proof they discussed in (9:11, p. 285).

Review Problems (pp. 148–154)

Time saved earlier in the chapter can be profitably spent on these problems.

Review Exercises, Chapters 1 to 5 (pp. 155–158)

The more subtle of these "true or false" questions should be discussed with the class. Misunderstandings and false ideas exist even among good students with a strong SMSG background.

Chapter 6: (10:6)

A CLOSER LOOK AT PROOF (pp. 159–188)

6-1 How a Deductive System Works (p. 159)

Understanding of what is involved in logical reasoning and setting up a mathematical theory comes slowly even for well-prepared students.

6-2 Indirect Proof (pp. 160–167)

Though indirect reasoning has been used in justifying properties developed in earlier SMSG texts and in proving that $\sqrt{2}$ is irrational in (9:7) and (J-2:6), it has not been labeled as such.

6-3 Theorems About Perpendiculars (pp. 167–175)

The idea of perpendicularity and construction of a perpendicular to a line are taught in (E-6:9) and (J-2:4); however, the proofs here are new. Section 6-3 should be taught as written.

6-4 Introducing Auxiliary Sets into Proofs (pp. 176–181) and

6-5 Betweenness and Separation (pp. 182–188)

This is all new material.

Chapter 7: (10:7)

GEOMETRIC INEQUALITIES (pp. 189-218)

7-1 Making Reasonable Conjectures (pp. 189–191)

The practice of making reasonable conjectures has been used extensively in SMSG materials, beginning as early as (J-1:1, p. 6) and (J-1:3, p. 77). Relationships between the sides and angles of a triangle were explored in (E-6:3) and (J-1:10).

7-2 Algebra of Inequalities (pp. 191–192)

It is well to review the inequalities of real numbers (9:8) before considering geometric inequalities.

7-3 The Basic Inequality Theorems (pp. 193–213)

Theorems 7-1 through 7-9 should be treated as new, although many exercises in the junior high material hint at their existence; for instance, Theorem 7-6 is stated as a property in (J-2:11).

7-4 Altitudes (pp. 214–216)

Altitudes of a triangle are investigated in (E-5:8), redefined and used in (J-1:10) and (J-2:11). Treat this as a review but do not omit.

Chapter 8: (10:8)

PERPENDICULAR LINES AND PLANES IN SPACE (pp. 219–239)

8-1 The Basic Definition (pp. 219–221)

The properties of lines and planes in space studied in (J-2:11) form a background for this chapter.

8-2 The Basic Theorem (pp. 222–235)

Theorem 8-3 was stated as a property in (J-2:11); otherwise the theorems in this section are new to the student.

8-3 Existence and Uniqueness Theorems (p. 235)

This section contains new material.

Chapter 9: (10:9)

PARALLEL LINES IN A PLANE (pp. 241–290)

9-1 Conditions Which Guarantee Parallelism (pp. 241–250)

Parallel lines are defined in (J-1:4, 10). The definition of a transversal is also given in (J-1:10). The relationship between parallel lines and alternate interior angles is discovered in (J-2:4). Treat as a formalization and extension of previous learning.

9-2 Corresponding Angles (pp. 251–252)

Corresponding angles are defined in (J-1:10). Theorem 9-7 is given as a property of parallel lines in (J-1:10).

9-3 The Parallel Postulate (pp. 252–258)

Theorem 9-8 is discovered in (J-2:4). Theorem 9-9 is stated as a property of parallel lines in (J-1:10). This section is review and extension.

9-4 Triangles (pp. 258–262)

Theorem 9-13 is discovered by using known properties of lines and angles in (J-1:10). Earlier, in (E-6:3), it was discovered by measuring.

9-5 Quadrilaterals in Plane (pp. 263–267)

Quadrilaterals were introduced in (E-4:5). Opposite sides, opposite angles, diagonals, and consecutive sides have been defined. A trapezoid is defined in (J-2:11). A parallelogram is defined in (J-1:10) and many of its properties discussed. Theorem 9-14 was proven as an exercise in (J-2:4). Theorems 9-15, 9-16, and 9-17 are given as properties of a parallelogram in (J-1:10). The distance between two parallel lines is defined in (J-1:10). Theorem 9-18 is used in an exercise of (J-2:4). Theorems 9-19 through 9-22 are new.

9-6 Rhombus, Rectangle and Square (pp. 268–274)

The definitions of rhombus, rectangle, and square have been given in previous SMSG texts (E-4:8), (E-5:8), (J-1:8), and (J-2:11). Theorems 9-23 through 9-25 are new to students.

9-7 Transversals to Many Parallel Lines (pp. 275–283)

Concurrent lines are defined in (J-2:4). Theorem 9-27 is discovered through an exercise in (J-2:4). Treat as a review and extension of concepts.

Chapter 10: (10:10)

PARALLELS IN SPACE (pp. 292–315)

10-1 Parallel Planes (pp. 292–299)

Parallel planes are discussed in (E-4:5), defined in (J-1:4) and (J-2:11). Theorem 10-2 appears in an exercise and Theorem 10-5 is stated as a property in (J-2:11). This section should be treated as an extension of previous concepts.

10-2 Dihedral Angles, Perpendicular Planes (pp. 299–306) and

10-3 Projections (pp. 306–312)

This is new material.

SMSG Geometry, Part II

Chapter 11: (10:11)

AREAS OF POLYGONAL REGIONS (pp. 317–357)

11-1 Polygonal Regions (pp. 317–327)

Polygonal regions are defined in (E-4:5) in discussion of triangles. Triangular and quadrilateral regions are again met in (E-5:8). Space regions are defined in (E-6:7) and (J-2:10). Postulate 20 has been used in (E-5:8), (J-1:7), and (J-2:11). Problem 8 is discussed in (J-2:10) and (J-2:13). This section unifies and extends such material.

11-2 Areas of Triangles and Quarilaterals (pp. 328–338)

Area of a triangle was introduced in (E-5:8) and met again in (J-1:10). Theorem 11-2 is used in (E-5:8) and (J-1:10) and stated as a formula in (J-2:11). Theorem 11-3 is stated in (J-1:10) and (J-2:11). Theorem 11-4 is stated and written as a formula in (J-2:11). Treat as review leading to extension.

11-3 The Pythagorean Theorem (pp. 339–353)

The Pythagorean theorem is stated and two proofs are given in (J-2:4). An extremely simple nonalgebraic proof, adapted to an overhead projector, appears in SMSG *Introduction to Secondary School Mathematics,* Volume 2, Part II, Chapter 19. Special triangles (Theorems 11-9 and 11-10) are treated in (E-6:3). Here these topics are amplified, formalized.

Chapter 12: (10:12)

SIMILARITY (pp. 359–403)

12-1 The Idea of a Similarity (pp. 359–364)

Similar triangles, not identified as such, were used to introduce ratio and proportion in (J-1:9). Proportions were then used in the balance of the chapter to solve percent problems. The ninth grade text contains but brief references to ratio and proportion (9:9, 15). The definition of geometric mean is new to students.

12-2 Similarities between Triangles (pp. 364–367)

Similar triangles are defined in (J-2:9). Treat this section as an extension of previous concepts.

12-3 The Basic Similarity Theorems (pp. 367–391)

The theorems are new; they draw on exercises in (J-2:9) discovering the principles of Theorems 12-3, 12-4, and 12-5.

12-4 Similarities in Right Triangles (pp. 391–394) and

12-5 Areas of Similar Triangles (pp. 395–400)

All new material.

Chapter 13: (10:13)

CIRCLES AND SPHERES (pp. 409–460)

13-1 Basic Definitions (pp. 409–412)

The student should have a broad background for the topics given new formalization here. Circles named by their centers and determined by their radii are studied in (E-4:5), (E-6:9), and (J-1:11). The arc and its center are presented in (E-5:4), (E-6:9), and (J-1:11). In (E-6:9) and (J-1:11) can be found definitions of chord, diameter, and central angle. The semicircle is included in (J-1:11). Theorem 13-1 appears in (J-2:12).

13-2 Tangent Lines. The Fundamental Theorem for Circles (pp. 412–422)

Tangents are mentioned in (J-1:11) and much ground work has been laid for Theorem 13-2. Some re-teaching is essential.

13-3 Tangent Planes. The Fundamental Theorem for Spheres (pp. 423–438)

Spheres have not been given extensive treatment, although they were mentioned in (E-4:8). (J-2:12), often omitted from the eighth grade curriculum, introduced great circles and tangent planes. This section (13-3) may be discussed intuitively, since it parallels very closely the previous section, (13-2).

13-4 Arcs of Circles (pp. 429–447)

Though the students have had some background from (E-6:9) and (J-1:11), most of this section will be new to them.

13-5 Lengths of Tangents and Secant Segments (pp. 448–456)

This material is new.

Chapter 14: (10:14)

CHARACTERIZATIONS OF SEGMENTS. CONSTRUCTIONS (pp. 461–504)

14-1 Characterization of Sets (pp. 461–463)

Sets have been an unifying concept throughout SMSG texts. This is a new approach.

14-2 Basic Characterizations. Concurrence Theorems (pp. 464–473)

In (E-6:9) and (J-2:4) students learned to do some compass and straightedge constructions. Foundation for Theorem 14-4 was laid in an exercise of (J-2:4).

14-3 Intersection of Sets (pp. 473–475)

The language and ideas are familiar but the approach here is new.

14-4 Construction with Straightedge and Compass (pp. 475–477)

The student had some experience with constructions in (E-6:9) and (J-2:4).

14-5 Elementary Constructions (pp. 477–489)

Constructions 14-7, 14-8, and 14-8-1 appear in (J-2:4). Construction 14-9 appears in both (E-6:9) and (J-2:4). The teacher needs to be aware of this background and present the material taking advantage of recent months of formal geometry.

14-6 Inscribed and Circumscribed Circles (pp. 490–493)

This is not essentially new material but the evelopment is.

4-7 The Impossible Construction Problems of Antiquity (pp. 493–502)

This material is new for most students, and sually promotes such a lively discussion that xtra time is needed.

Review Problems (pp. 503–504)

There are few problems here; if there is time, he student should be provided with additional eview problems before proceeding.

hapter 15: (10:15)

REAS OF CIRCLES AND SECTORS (pp. 505–531)

5-1 Polygons (pp. 505–509) and

5-2 Regular Polygons (pp. 510–515)

This section builds on the formula for the um of the angles of an *n*-sided polygon from (E-6:3) and the regular polygons studied in (J-2:11).

5-3 The Circumference of a Circle. The Number π (pp. 516–519) and

5-4 Area of a Circle (pp. 520–524)

In (J-1:11) circular regions are discussed and here is some attempt at that point to arrive at the alue π through measurement, but the formula for circumference of a circle is essentially assumed. In (J-1:10) the formula for the area is simply stated, not rationalized as the elaboration on (J-2:11) is here.

15-5 Lengths of Arcs. Areas of Sectors (pp. 525–529)

Length of an arc and area of a sector discussed in (E-2:11) as part of investigations of the lateral area of a cone, are extended in this section.

Review Problems (pp. 530–531)

In reviewing the chapter, supplementary problems like number seven and number seventeen add interest.

Chapter 16: (10:16)

VOLUMES OF SOLIDS (pp. 533–565)

16-1 Prisms (pp. 533–539),

16-2 Pyramids (pp. 540–545),

16-3 Volumes of Prisms and Pyramids,
Calvalieri's Principle (pp. 546–553),

16-4 Cylinders and Cones (pp. 553–559), and

16-5 Spheres: Volume and Area (pp. 559–563)

This material is not totally new to the students. Prisms, altitude, cross-section, lateral edge face, and surface were studied in (E-4:8), (E-6:7) and (J-2:11); parallelepipeds, in (J-2:11). Pyramid have been met in (E-4:8), (E-6:7), and (J-2:11).

Formulas in Theorems 16-7, 16-9, and 16-10 ar stated and used in (E-6:7). Theorem 16-7 and Theorem 16-10 are in (J-2:11). Theorem 16-15 is presented in (J-2:11). The volume of a circular cylinder is discussed in (J-1:11). The volume and surface area of a sphere occur in (J-2:12). All of this introductory work was done without proofs. Trea the present sections as extensions on the familiar ideas.

Chapter 17: (10:17)

PLANE COORDINATE GEOMETRY (pp. 567–629)

17-1 Introduction (p. 567)

It is to be hoped that teachers have taken enough advantage of students' background in intuitive geometry to reach this important final chapter before the close of school. The entire chapter is really too brief an introduction, and would be enhanced by supplementation from SMSG *Geometry with Coordinates,* Part II.

17-2 Coordinate Systems in a Plane (pp. 567–572)

17-3 How to Plot Points on Graph Paper
(pp. 572–575), and

17-4 The Slope of a Non-Vertical Line
(pp. 576–583)

Essential review of (E-6:5), (J-2:1), (J-2:9), (9:14), and (9:15).

17-5 Parallel and Perpendicular Lines
(pp. 583–587)

Theorem 17-2 appears in (9:14) but Theorem 17-3 is new.

17-6 The Distance Formula (pp. 588–591)

This is more formalized than any presentation previously.

17-7 The Mid-Point Formula (pp. 592–595)

Students find the formula intuitively simple to understand, but the proof, which is new, is more difficult for them.

17-8 Proofs of Geometric Theorems (pp. 595–600)

This is new and difficult, and requires more time than it appears. Expansion is desirable.

17-9 The Graph of a Condition (pp. 600–604)

The students have been well prepared for this section in the graphing of the junior high school texts and in (9:14, 15). Some of the more difficult problems from the ninth grade text might well be used as supplementary work here.

17-10 How to Describe a Line by an Equation (pp. 604–611)

This review of (9:14) is essential here.

17-11 Various Forms of the Equation of a Line (pp. 611–613)

Theorem 17-8 appears in (9:14), but the students need this new explanation.

17-12 The General Form of the Equation of a Line (pp. 613–616)

Though Theorem 17-10 appears in (9:14), the teacher must not assume that the students remember it or can use it.

17-13 Intersection of Lines (pp. 617–621)

The intersection of lines found by graphing and by the algebraic solutions of two linear equations appear in (9:15). The vocabulary here is slightly different.

17-14 Circles (pp. 621–628)

This is new material.

SCHOOL
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ARTICULATION OF CONTENT OF SMSG TEXTS GRADES 1-3 and GRADE 4

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ARTICULATION OF CONTENT OF SMSG TEXTS FOR GRADES 1–3 and GRADE 4

In the development of the sample texts for pupils and commentaries for teachers which comprise SMSG's *Mathematics for the Elementary School,* the materials for grades 4–6 preceded the materials for grades K–3. Within each of these two levels—intermediate grades and primary grades—the content sequence was sufficiently well articulated. This is not as true, however, for the content articulation between Books 3 and 4. And for the following reason:

The texts for the intermediate grades necessarily were written for pupils who had no comparable kind of background previously in the primary grades. Thus, a portion of the material for grade 4 presented to pupils a *new view* of some of the mathematical content that had been studied from a traditional point of view and in a narrowly limited way during earlier grades.

When in due course of time text materials were developed from a contemporary point of view for grades K–3, a certain lack of articulation between Books 3 and 4, in particular, was inevitable. It was deemed better to face this situation honestly than to try to avoid it by having the content of Book 4 place an artificial delimitation upon the content to be included in Books K–3.

This present report, prepared by J. F. Weaver with the assistance of Carole Greenes and Priscilla Greer, is similar in its intent to the report on "Articulation of Content of SMSG Texts, Grades 7–10" (*Newsletter* No. 25; December 1966).

In this instance, however, we concentrate on the articulation of text content for grades 3 and 4, with some reference to grades 1 and 2 wherever appropriate and helpful. The material presented in this report should facilitate greatly the transition from the use of SMSG texts in grades 1–3 to the use of SMSG texts in grades 4–6, with emphasis upon a smoother articulation of content for grades 4 following the sequence for grades 1–3.

In this report, references to SMSG texts appear in the following forms: 3: IV-5 (grade, chapter, section), or 3: IV (grade, chapter), or IV-5 (chapter, section). Page numbers, where indicated, refer to the *Teacher Commentary,* not the Student Text.

GRADE 4

Chapter 1

CONCEPT OF SETS (pp. 1–50)

All conceptual ideas in this chapter have been introduced and used on numerous occasions in Books 1, 2 and 3. This material is essentially a review with these two extensions:

I-5. Union of Sets (pp. 29–35)

The *symbol* for set union, ∪, is new content.

I-6. Intersection of Sets (pp. 36–42)

The *symbol* for set intersection, ∩, is new content. Note that the *concepts* of union and intersection are *not* new.

Chapter 2

NUMERATION (pp. 51–109)

II-1. Grouping in Base Five (pp. 60–68)

II-2. Base-Five Notation (pp. 69–75)

II-3. Grouping and Notation in Other Bases (pp. 76–81)

This is the first time that explicit consideration has been given to a scheme of place-value notation having a base other than ten. This should be developed as new content. It is not intended that pupils become skillful in this work; rather, that they gain some appreciation of *base* and its significance in connection with place-value notation.

II-4. A Number May Have Several Names (pp. 82–85)

II-5. Renaming Numbers (pp. 86–91)

II-6. Extending Ideas of the Decimal System (pp. 92–98)

II-7. Order Relations on the Number Line (pp. 99–105)

II-8. Some New Symbols (pp. 106–108)

II-9. Just for Fun (p. 109)

All concepts and symbolism used in these sections have been developed progressively during the course of Books 1, 2 and 3. Basically this is review material, except that some extensions are made beyond 4-place numerals considered in Book 3, to 5- and 6-place numerals here in grade 4.

Chapter 3

PROPERTIES AND TECHNIQUES OF ADDITION AND SUBTRACTION I (pp. 111–228)

III-1. Addition and Subtraction (pp. 117–123)

This essentially reviews the interpretation of addition in relation to the union of disjoint sets, and should be a very familiar idea to pupils.

III-2. Addition and the Number Line (pp. 124–128)

An interpretation of addition in relation to the number line was made very explicit in (2:II-1), but received less detailed consideration in Book 3. Although this is essentially review, this interpretation of addition is an important one and should not be slighted.

III-3. Addition and Subtraction as Operations (pp. 129–133)

This is an overview of a higher level interpretation of addition and subtraction (and multiplication and division) as *operations*. Treat the essential nature of this idea as new material.

III-4. True Mathematical Sentences (pp. 134–142)

Mathematical sentences, and number sentences in particular, should be familiar to pupils from Books 1, 2 and 3. However, this section makes much more explicit than before the fact that a sentence may be true or it may be false. Treat the material accordingly.

III-5. Thinking About Addition Facts (pp. 143–150)

This is clearly review material.

III-6. The Commutative Property for Addition (pp. 151–156)

This property has been developed intuitively in preceding grades. This is the first time, however, that the property has been made this explicit and given a name (commutative).

III-7. Thinking About Subtraction Facts (pp. 157–166)

Although subtraction facts are familiar to pupils, here subtraction is interpreted explicitly in terms of finding an unknown addend. Although this interpretation has been implied in earlier grades, you will need to deal carefully with this interpretation in this section.

III-8. Mathematical Sentences Using the Number Line (pp. 167–169)

III-9. Number Line Pictures and Mathematical Sentences (pp. 170–172)

These sections review briefly section III-2 of this chapter and extend the work to interpret subtraction in relation to the number line.

III-10. More Mathematical Sentences (pp. 173–178)

III-11. Using Mathematical Sentences in Problem Solving (pp. 179–189)

These related sections provide higher level interpretations of addition and subtraction, as have been developed during the course of this chapter. The emphasis is upon addition as an operation for finding the sum when two addends are known, and upon subtraction as an operation for finding the unknown addend when a sum and one addend are known. The application of these interpretations, and of mathematical sentences, to problem solving situations should be treated carefully and in some depth.

III-12. Doing and Undoing — Addition and Subtraction (pp. 190–197)

This is a review of material introduced prior to this grade, but the treatment is more explicit than before.

III-13. More About Addition and Subtraction of Whole Numbers (pp. 198–206)

Although the ideas of this section have been glimpsed implicitly in earlier work, this material should be handled essentially as new content: an explicit recognition of the closure property for addition within the set of whole numbers (and the lack of this property in the case of subtraction within the set of whole numbers).

III-14. More Problem Solving (pp. 207–212)

This extends the work of section III-11 of this chapter.

III-15. The Associative Property for Addition (pp. 213–220)

Although this property has been used in an implicit way in preceding grades, this is the first explicit statement of the property. The material should be treated accordingly.

III-16. Review (pp. 220–228)

Chapter 4

PROPERTIES OF MULTIPLICATION AND DIVISION (pp. 229–386)

These operations, and properties associated with them, have been developed extensively in Books 1, 2 and 3:

1: VIII – Arrays and Multiplication
2: VIII – Arrays and Multiplication
2: IX – Division and Rational Numbers
3: IV – Arrays and Multiplication
3: VII – Multiplication, Quotients, and Division
3: IX – Division

In light of this extensive background, there are very few instances in which the material of this chapter is not strictly review and may be treated accordingly. In addition to a more explicit interpretation of division as an operation for finding a missing factor when a product and one factor are known, the following sections should be noted:

IV-6. Using the Number Line (pp. 287–289)

This interpretation of multiplication may be treated as new content.

IV-10. Closure (pp. 313–316)

This makes more explicit the closure property as it does or does not apply for the operations of multiplication and division within the set of whole numbers.

IV-12. Distributive Property of Division Over Addition (pp. 333–337)

Treat this as new content.

IV-15. Partitioning Sets (pp. 353–356)

IV-16. Making a Record of Our Thinking (pp. 357–362)

The ideas presented in these two sections are not new. However, pupils encountered two things in connection with their division work in Book 3 that are not in any way reflected in the division work in Grade 4:

(1) an expression such as "$32 \div 4$" also was symbolized as $\frac{32}{4}$. Each expression was read in the same way: "32 divided by 4." (See 3:VIII)

(2) Because of more extensive work with rational numbers in Book 3, the quotient for a division such as $32 \div 5$ was expressed as $6 + \frac{2}{5}$; that is, $32 \div 5 = 6 + \frac{2}{5}$. Pupils who have used Book 3 are more familiar with this way of handling "remainders" than with the way used here—in which the following sentence is associated with "$32 \div 5$": $32 = (6 \times 5) + 2$. This latter form is appropriate here, but you also may wish to have children recall the way in which they handled remainders in the Book 3 approach. (See 3:IX)

Chapter 5

SETS OF POINTS (pp. 387–491)

Extensive work with non-metric geometry has preceded this chapter; e.g.,

1: V — Recognizing Geometric Figures
2: III — Sets of Points
2: VII — Congruence of Angles and Triangles
3: I — Sets of Points

The material here in Chapter 5 can, for the most part, be developed as a review of ideas with which children already have some familiarity. However, the approach and organization are somewhat different at times from that used in Books 1, 2 and 3. This has an obvious motivational advantage.

The following sections of this chapter are the ones that involve either new material or an explicit extension or higher level interpretation of material:

V-7. Planes (pp. 430–436)

V-8. Lines and Planes (pp. 437–444)

V-9. Intersection of Lines and Planes (pp. 445–452)

V-12. Circles (pp. 465–472)

V-13. Regions in a Plane (pp. 473–477)

V-15. Angles of a Triangle (pp. 486–491)

Chapter 6

PROPERTIES AND TECHNIQUES OF ADDITION AND SUBTRACTION II (pp. 493–563)

This chapter is devoted to the development of further skill in computing, using addition and subtraction, and to applications of such skill. The techniques presented, and the principles upon which they are based, are no different than those involved in (3: V). Now, however, numbers greater than 999 are used as addends – extending to those named by 4- and 5-place numerals.

The only relatively new or novel content is in these sections:

VI-11. If-Then Thinking (pp. 547–550)

This makes explicit a kind of relational thinking that pupils have used informally and implicitly before.

VI-13. Enrichment (pp. 558–562)

This extends, on an optional basis, the interpretation of a mathematical operation.

Chapter 7

TECHNIQUES OF MULTIPLICATION AND DIVISION (pp. 575–692)

In the preceding year, in addition to "basic facts," pupils should have developed skill in multiplication and division to the following extent:

Multiplying and dividing with multiples of 10 (3: VII)

Multiplying numbers between 10 and 20 by numbers less than 10 (3: VII)

"Division with remainders," as delimited by use of basic facts (3: IX)

Also, pupils should be familiar with an algorithm for the division process and with these forms of notation associated with division:

$$23 \div 7 = \frac{23}{7} \text{ and } \frac{23}{7} = 3 + \frac{2}{7}.$$

[Neither of these forms is used in Book 4, however.]

It also is true that pupils in Book 3 (3: IX) deal with situations in which they indicate whether, for instance, $\frac{3}{8}$ of 48 is greater than, equal to, or less than, $\frac{4}{7}$ of 42. [This kind of work is not considered in Book 4.]

The following sections of this chapter include extension of skill in computing with multiplication and division, using numbers greater than those used in earlier years; consequently, a degree of "new work" is present in each of these sections:

VII-2. Multiplying by Multiples of Ten (pp. 582–586)

VII-3. Multiplying by Multiples of One Hundred (pp. 587–591)

VII-4. More About Multiplying (pp. 592–598)

VII-5. Multiplying Larger Numbers (pp. 599–604)

VII-6. A Shorter Method of Multiplying (pp. 605–608)

VII-7. Multiply Numbers Less Than 100 by Multiples of 10 (pp. 609–613)

VII-8. Finding Products of Any Two Numbers Greater Than Ten (and Less Than One Hundred) (pp. 614–623)

VII-9. Using Multiplication in Problem Solving (pp. 624–627)

VII-10. Finding Unknown Factors (pp. 628–637)

VII-11. A Way of Dividing Two Numbers (pp. 638–641)

VII-12. More About Dividing Two Numbers (pp. 642–645)

VII-13. Using Division in Problem Solving (pp. 646–651)

VII-14. Becoming More Skillful in Dividing Numbers (pp. 652–659)

VII-15. Finding Quotients and Remainders (pp. 660–666)

VII-16. Reviewing and Extending (pp. 667–692

As undoubtedly is evident, this is a "meaty" chapter. It often is handled most effectively whe spread over an extended period of time, interspersed with geometric content.

Chapter 8

RECOGNITION OF COMMON GEOMETRIC FIGURES (pp. 693–766)

In light of the extent of work in non-metric geometry in Books 1, 2 and 3 that was mentioned earlier, much of this material is an extension and refinement of that work and is not "new" in a strict sense. It would be well to note particularly the following things, however.

VIII-1. Review the Triangle and Quadrilateral (pp. 703–707)

Note the use of "vertex" and "vertices" as applied to polygons.

VIII-3. Isosceles and Equilateral Triangles (pp. 713–718)

Note the construction involving compass and straightedge.

VIII-4. Right Angles (pp. 718–723)

Note the comparison of angle sizes that is included in this section.

VIII-5. Rectangles and Squares (pp. 724–727)

Note the explicit statements of relationship between rectangles and squares.

VIII-6. Surfaces (pp. 728–766)

You may develop this as essentially "new" material.

Chapter 9

LINEAR MEASUREMENT (pp. 780–841)

As with several other aspects of content, pupils have had considerable experience with linear measurement in Books 1, 2, and 3:

1: X — Linear Measurement
2: V — Linear Measurement
3: VI — Length and Area

This chapter can be developed principally within a review context, with no new mathematical ideas being introduced. But note in particular the following things:

IX-4. Measuring a Segment (pp. 792–794)

Note the clear distinction that is made in connection with these expressions or terms: "unit of measure," "a measure as a number," and "length."

IX-7. Using Standard Units of Length (pp. 800–807)

Note the use of the centimeter as a unit.

Although all of this work should appear familiar to pupils, it is presented at a somewhat higher level of approach than was true in preceding years.

Finally, notice that in (3: VI) pupils were introduced to the concept of the area of a plane region, but such work is *not* included in the text content for Grade 4. Be certain in connection with sections (IX-11) and (IX-12) of this chapter that pupils do not confuse the concept of area with that of perimeter.

Chapter 10

CONCEPT OF RATIONAL NUMBERS (pp. 845–934)

First note the frequency with which ideas regarding rational numbers have been developed in Books 1, 2 and 3:

1: IX – Partitions and Rational Numbers
2: IX – Division and Rational Numbers
3: VIII – Rational Numbers

In this preceding work the non-negative rational numbers have been associated with: (1) partitioning regions into congruent regions, (2) partitioning sets into equivalent sets, (3) partitioning segments into congruent segments, (4) points on the number line, and (5) the operation of division. Clearly, this background is more than adequate to make the most of this chapter (4: X) essentially review in nature.

There is one section of this chapter, however, for which a special note is in order:

X-6. A New Kind of Name (pp. 911–917)

Previously in (3: IX) pupils were introduced to statements of equality such as: $\frac{13}{3} = 4 + \frac{1}{3}$; but there was no explicit attempt made to shorten this to the following form: $\frac{13}{3} = 4\frac{1}{3}$. This abbreviated form is now used in this section, and an excellent background for its interpretation has been given through the work in (3: IX), with which pupils should be familiar.

CONCLUDING NOTES

In the preceding outline we frequently have indicated that topics in Grade 4 can be developed within a "review context" because of the rather extensive treatment which preceded in Books 1, 2 or 3.

You will find, however, that the level of presentation and the level of approach are appropriately higher in Grade 4 than for Books 1, 2 and 3. This can be a distinct advantage in that familiar content is not simply "rehashed" for pupils, but is truly *re-viewed:* viewed in a light which brings content into a sharper focus than was true in earlier years. Do all that you can to help pupils sense this fact as they work anew with ideas that, in essence, are familiar to them.

Pupils will bring to Grade 4 an uncommon understanding of what we call the whole-number line in light of their work in Book 3 with (3: III): Describing Points as Numbers. You can use this background to good advantage by making more frequent use of the number line that is indicated specifically in the text material for Grade 4.

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September 1970

MATHEMATICS FOR DISADVANTAGED AND LOW ACHIEVING STUDENTS

Early in its history, SMSG demonstrated that t was not concerned solely with above average college-bound students. The first junior and senior high school SMSG texts were designed for college-capable students, but even before these were finished work had been started on a revision of the texts for grades 7, 8, and 9, which would be more useful for average and below average students than the original versions. These texts, INTRODUCTION TO SECONDARY SCHOOL MATHEMATICS, Volumes 1 and 2, and INTRODUCTION TO ALGEBRA, were finished n 1962.

* * *

During 1963, arrangements were made for a tudy to be carried out during the following two cademic years in which these texts were used by tudents between the 25th and the 50th percentile n general ability. This study demonstrated that uch students could learn about as much mathematics as above average students provided they were given more time. Further information on his study is contained in SMSG Report No. 5.

* * *

In 1964, in order to obtain comments and suggestions from the mathematical community, MSG convened a conference to discuss all aspects f mathematics education for below average chievers. A report of this conference was published.

* * *

As a result of this conference, SMSG took on wo new activities. One was concerned with primary school mathematics for disadvantaged children and led to a revision of the SMSG materials or kindergarten and grade one.

Two assumptions were made in revising the riginal SMSG books to make them more useful or children with backgrounds that differ significantly from what is usually thought of as "middle nd/or upper class."

1. These children are entitled to a mathematics curriculum which is mathematically sound and properly sequenced from both a mathematical and pedagogical point-of-view.
2. They can learn from such a curriculum if

> the material is presented in a way whic
> takes into account the presence or absenc
> of the special skills, attitudes, and behavio
> which the school seems to demand as a pre
> requisite for academic success.

Since the existing books had been revised i
light of teacher reports during a year try-out, on
site observations and the recommendations of con
sultants, the revised editions met the requiremen
of the first assumption.

To find out if the material was appropriate fo
disadvantaged children and if not, how it migh
be modified for maximum benefit, SMSG set u
centers in Boston, Chicago, Detroit, Miami, Oak
land (California) and Washington, D. C. Seve
kindergarten and eight first grade teachers an
their classes were involved. The designation of
particular school in the disadvantaged area o
each city, as well as the selection of teachers an
their classes were made by the local school system

The fifteen teachers were provided with all th
available educational and play materials suitabl
for their classes as well as the revised SMSG texts
Each teacher had available the services of a mathe
matics consultant and a psychologist at regula
intervals.

The teachers made weekly reports describin
and evaluating their daily mathematics lesson
and following the progress of individual students
They prepared reports on each chapter of th
SMSG books as they completed the material. Th
teachers also met, as a group, with committe
members of SMSG four times during the schoo
year to discuss progress, report difficulties and t
recommend modifications of the existing SMS
text materials. All classes were visited at least onc
by a member of the SMSG staff at Stanford Uni
versity.

As a result of the teachers' reports, the grou
meetings, and the on-site observations and recom
mendations of mathematics consultants and psy
chologists, the existing SMSG kindergarten an
first grade texts were rewritten. They are no
published under the same title followed by "Spe
cial Edition."

The Special Edition of the kindergarten boo
reflects teacher concern that many different activi
ties and types of material be readily at hand t
introduce and reinforce the concepts which ar
considered preparatory to first grade work. Teach
ers are encouraged to look upon this period as on
which is exploratory and developmental. Master
of concepts by every child is NOT implied.

The Special Edition of grade one begins with a review of the concepts presented in kindergarten but provides a complete teaching procedure so that the child without kindergarten experience is not penalized. Many more student pages are available where teachers have indicated a need for a slower pace in the development of a particular concept. In some cases the order of presentation was altered. For example, teachers found it easier to teach the numbers 11–19 after ten and the multiples of ten less than one hundred had been developed rather than stress the numbers 20–99 and then return to the teens. Those pages which are to be used in the teaching of a lesson are so marked. In the teacher edition many activities which stress the use of manipulative materials are suggested before a given page in the book is to be considered. If the concept is a difficult one, such as the idea of "fewer than," for ALL children, the teacher is alerted to this fact in order to avoid frustration at the lack of immediate results. With these children the pace will be slower and the range of achievement is apt to be greater than in a middle class heterogeneous group. Consequently, teachers are made aware of situations which will require a great deal of development and the necessity of making additional worksheets to fit the individual differences in the class. Unhappily for teachers, there is NO book of any kind or description which has enough pages of varying difficulty to fit the range of needs in any one given class.

For the benefit of teachers of primary school disadvantaged children, a new volume, No. 13, was added to the *Studies in Mathematics* series. This book was written at the request of teachers and consultants who indicated a need for a book devoted to the mathematics and pedagogy of a contemporary instructional program that emphasizes conceptual learning. The beginning chapter discusses the factors which contribute to being disadvantaged, a description of such children and the implications for teaching. The following seventeen chapters introduce and provide a progressive development of significant ideas in the K-3 program. In each chapter the exposition of a concept and the related development of appropriate skills is followed by a section called "Applications for Teaching." This unusual feature is an important one since the mathematics dictates the pedagogy. The reader is told what ideas, language, etc. may prove difficult for disadvantaged children and provides suggestions which other teachers have found helpful. Before the usual listing of new vocabulary and exercises for each chapter is a small section entitled "Question." The question or questions considered here are the ones most frequently raised by teachers of disadvantaged children when working with the mathematics consultant. They highlight the points which seem to cause teachers the greatest amount of misunderstanding and erroneous interpretation.

The appendices deal with (1) the scope and organization of the K-3 program, (2) the usefulness and difficulties of the language of mathematics and (3) a report of the study of disadvantaged children which supplied the information needed to write the Special Editions and this volume.

SMSG Reports 2 and 4 provide further information on these activities.

A by-product of these activities was a booklet meant for nursery school teachers who are working with disadvantaged children. A number of mathematical ideas which are normally explored in kindergarten or the first grade are described. For each of these, a number of nursery school activities are suggested which should facilitate the student's later development of these topics in kindergarten or first grade. All the activities suggested were tried out with disadvantaged nursery school children and found to be feasible.

* * *

A second SMSG activity undertaken as a result of the 1964 conference is now culminating in the publication of a junior high school mathematics program SECONDARY SCHOOL MATHEMATICS – SPECIAL EDITION designed for students whose mathematics achievement in elementary school was very low. The first nine chapters, more than enough for one school year, were made available in September 1970 and another year's worth of material will be made available the following September.

The mathematical content of this junior high school program is derived from the new SMSG Secondary School Mathematics Program, described originally in SMSG Newsletter No. 24 and again more briefly in Newsletter No. 30. However, the format of this special edition is a decided departure from that of the usual classroom textbook.

This change of format comes as a result of a small experiment, conducted over a three-year period, with junior high school very low achievers in mathematics. Details of this experiment will be found in SMSG Reports 6 and 7. The heart of this experiment was the development of materials that would relieve the student from the burdens of computation, as much as possible, and concentrate on mathematical concepts and relationships. Tables were provided to enable the student to compute whenever the content of the material so dictated. A second departure from the norm was that students were not provided with textbooks but were issued daily worksheets on which the lesson for the day was printed and which contained ample space to do any of the required work. Worksheets were placed in a binder which, in most cases, was kept in the classroom. There were two factors that prompted this approach. First, because of these students' apparent immaturity and lack of organization, a textbook became a handicap to them. Simply keeping track of its physical location appeared to be beyond the capabilities of most of the students. Secondly, this approach seemed to produce the more positive effect of having the students consider what they had accomplished, rather than projecting what they had yet to do. These two aspects of the experiment proved to be extremely successful both from the point of view of the student and the teacher. The students included in this experiment learned about as much mathematics as similar students in control classes; however, there were marked attitude changes among the experimental students which did not appear among the control students and these changes were deemed to be desirable ones. In addition, discipline problems were markedly lower in the experimental classes than in the control classes. SMSG has retained these features in the development of this new material.

A group of writers prepared experimental versions of nine chapters during the summer of 1969. During the 1969–1970 school year, sixteen seventh grade classes, taught by fourteen teachers in eleven different schools, tried out the experimental chapters. All the students were low achievers in mathematics. Several classes consisted of Black students and two of Mexican-American students. Most of the classes finished eight chapters. The reactions of the students, the parents, and the teachers were extremely favorable. Teacher evaluations for each chapter were systematically collected and these evaluations served as a basis for the revision of the nine chapters during the summer of 1970.

Throughout the trial period participating teachers attended biweekly seminars in which the materials and teaching problems were discussed. Problems which arose were carefully noted and ways and means of counteracting them have been incorporated in a short teachers commentary. Suggestions for handling the material have also been included in the commentary.

The following are chapter headings of the material available:

Revised Version

1. Flow Charts
2. Structuring Space
3. Functions
4. Number Theory
5. The Integers
6. Rational Numbers
7. Probability
8. Equations
9. Congruence

It should be noted that although this material is presented to the student in the form of work sheets, it is not, in the ordinary sense, a workbook. Topics in each chapter are sequentially developed lesson by lesson. Teacher-led class discussion exercises are carefully programmed to lead the student to successful experiences in the exercise sets. Every effort has been made to construct lessons that prevent failure. The quantity of reading and the reading level have been reduced to a minimum. At the end of each chapter there are included: (1) a cumulative "self-test," which enables the student, on his own, to determine how he is progressing; (2) a practice test, which in essence, tells the student what he is expected to know; and (3) a chapter test which is administered by the teacher.

As the cost of producing individual worksheets commercially proved to be prohibitive, this material will come to the teacher in bound volumes. Each chapter will consist of a number of lessons of one or more pages. Each page will be perforated for easy removal. These pages can then be reproduced in quantities sufficient for the class by means of spirit master units. Although this reproduction process may appear to place an extra burden upon the teacher, in practice the extra time required to reproduce the material is more than compensated for by the positive results, both mathematically and behaviorally, that appear in the classroom.

* * *

Finally, early in 1970, SMSG convened another conference to discuss mathematics programs in inner-city schools. During this conference, most of the more promising inner-city school mathematics programs were reviewed and discussed, and a number of suggestions were presented for ways of improving these programs. A report of this conference is available.

* * *

Obviously, SMSG has merely scratched the surface of the problem of providing suitable mathematics programs for disadvantaged and low achieving students. We hope, however, that the activities will point the way to more numerous and more powerful efforts in the future.

REPORTS

This series consists of reports, too long to be included in SMSG Newsletters, on various SMSG projects. Single copies may be obtained by a postcard request to SMSG, Cedar Hall, Stanford University, Stanford, California 94305.

2. *The Special Curriculum Project*
4. *The Special Curriculum Project: 1965–66*
5. *The Slow Learner Project: The Secondary School "Slow Learner" in Mathematics*
6. *Preliminary Report on an Experiment with Junior High School Very Low Achievers in Mathematics*
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Newsletter No. 36

February 1972

FINAL REPORT ON A NEW CURRICULUM PROJECT

FINAL REPORT ON A NEW CURRICULUM PROJECT

oals of the Project

From 1958 to 1966, SMSG devoted the major rt of its efforts to the preparation of sample xtbooks. Of course many other activities were o undertaken – preparation of materials for ichers and for the more able students, evalua- ns of various sorts, etc. – but all these together present less time and manpower than the text- ok effort. With the revision of a computer text ring the summer of 1966, a complete set of xts was available covering the entire range from ndergarten to the end of high school.

In 1966, upon the direction of its Advisory ard, the School Mathematics Study Group ini- ted the development of a new flexible, sequen- l program in mathematics for grades seven rough twelve. The primary emphasis was de- ted to developing a new junior high curriculum, *condary School Mathematics* (SSM), which capa- e students can complete within three years, ile less capable students may profitably spend ir or more years completing it. New ideas and ncepts are refined and generalized in a spiral nner throughout this sequence.

Distinguishing characteristics of SSM are:

this curriculum is devoted solely to those mathematical concepts which the SMSG planning and writing groups believe all citizens should know in order to function effectively in our society;

the usual grade placement of mathematical topics is ignored; instead, topics from arithmetic, algebra, and geometry are introduced in a logical sequence and in such a way as to provide mutual support;

certain topics new to the junior high school curriculum are included; in particular, functions, coordinate geometry, rigid motions, computer mathematics, probability, and statistics;

a strong attempt is made to make clear to the students the relevance of mathematics to problems of the real world.

Since the aim of SSM was to include those con- pts which all citizens should know, some topics ich are commonly taught in the first year of ebra are purposely postponed. These topics, formal deductive development of geometry, d topics from Intermediate Mathematics are ated in a follow-up course, *Secondary School* *vanced Mathematics* (SSAM), which is de-

signed for students who elect to take addition
mathematics after completing the junior hi
sequence. SSAM prepares the student for a varie
of courses: Analytic Geometry; Elementary Fu
tions and Calculus; Algorithms, Computatic
and Mathematics; Matrix Algebra; and Probab
ity and Statistical Inference.

Development of the Curriculum

The development of the new sequence beg
with letters sent to *Science* and to *The Mathem*
ics Teacher early in 1966 which announced t
new project and urged all those interested in wh
mathematics is taught in our schools to subm
suggestions and comments. Several hundr
thoughtful letters were received and studied
those involved in the project.

During the summers of 1966 and 1967, a gro
of mathematicians and secondary school teache
wrote detailed outlines of the proposed progra
for grades seven through nine and discussed
ternative proposals for curricula for grades t
through twelve. These outlines were based on t
recommendations of a panel of university mat
maticians, applied mathematicians, and pract
ing secondary school teachers, which had met
the spring of 1966 in New Orleans.

A small group of writers prepared experimen
versions of fourteen chapters during the 1966–
academic year based on the detailed outlin
Parts of several chapters were rewritten after t
materials had been tried in seventh grade cla
rooms. During the 1967–68 school year these e
perimental chapters were tried out in sixteen se
enth grade classes. Most of the classes finished b
tween ten and twelve of the proposed chapters.
general, the reactions of the students, the paren
and the teachers were quite favorable. Teach
and student evaluations were systematically c
lected and these evaluations served as a basis f
the revision of nine of the chapters during t
summer of 1968.

During the academic year 1967–68 anoth
group of writers prepared experimental versio
of fourteen more chapters. These chapters we
tried out by the same group of students and teac
ers in the eighth grade during the 1968–69 scho
year. The same group of teachers and anoth
group of seventh grade students tried out pr
liminary versions of the first twelve chapters du
ing the 1968–69 school year.

During the summers of 1969, 1970, and 197
other writing teams met to continue making r
visions based on teacher and student evaluatio

the versions used during the previous academic
ar. Classroom tryouts continued during the aca-
mic years of 1969–70 and 1970–71. New teach-
were involved in the project as the students
oved from junior high to high school. By the
d of 1970, the first eight chapters were pub-
hed in their final form. The remaining twenty
apters were published in their final form in
71.

The following people participated in planning
writing sessions for SSM and SSAM.

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G. Begle, School Mathematics Study Group
ax S. Bell, University of Chicago
vid Blakeslee, San Francisco State College
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nes C. McCaig, Cupertino School District,
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George Richardson, School Mathematics Stud
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Gail S. Young, Tulane University
Martha Zelinka, Weston High School, Weston
Mass.

Description of the Curriculum

The junior high sequence, SSM, has the foll
ing features.

1. An attempt is made to fuse arithmetic, a
bra, and geometry in such a way that each
helps in supporting the development of the ot
whenever possible.

2. Geometry is presented in a concrete, in
tive, descriptive way. When a new concept
introduced, the focus is on its essential featu
Its more complex aspects and its relation to ot
concepts are treated later. The treatment of
ometry fosters understanding of the concrete b
and the intuitive significance of geometric id
before they are studied on a rather formal ded
tive basis.

3. One, two, and three dimensions are c
sidered in many geometrical questions wh

pplicable. Coordinate geometry appears as ap-
ropriate in helping describe sets of points alge-
raically. Solution sets of algebraic equations are
nterpreted or described geometrically.

4. The process of model building in applied
athematics is the construction of a mathematical
odel that will help one better understand a situ-
tion that occurs in a complex environment, since
e analysis of such a mathematical situation
ften enables one to learn more about the origi-
al physical situation. This process is developed
appropriate places in the sequence.

5. Some relaxation in the stress on structure is
oticeable, *but* structure is still definitely one of
e unifying themes.

6. Short deductive sequences appear using prop-
rties discovered in the intuitive development of
oth algebra and geometry. Various methods of
roof are illustrated.

7. Appropriate materials are included so that
e student has some appreciation for the mathe-
atical activities associated with computer use
nd the role of computers in our technological
ciety. Mathematical algorithms expressed in
ow chart notation offer an introduction to com-
ter activity and require the student to have
fficient understanding of the mathematics for
ractical exploitation.

8. The concept of function is considered early
nd used in many different types of mathematical
ntent whenever appropriate.

9. The development of geometry culminates
ith a chapter on rigid motions and vectors.

10. Three chapters introduce probability and
atistics and lead up to some examples of hy-
othesis testing.

As with all SMSG texts, each chapter is ac-
mpanied by a Teacher's Commentary. In this
quence, each commentary describes why the
apter was included and how it fits with the
her chapters, states the purposes and objectives
the chapter, gives a suggested time schedule,
ovides additional background for each section
d answers to all exercises, and concludes with
ggested test items and answers.

The SSM sequence is published in fourteen
its with two chapters to a unit. The chapter
les are:

1. Structuring Space
2. Functions
3. Informal Algorithms and Flow Charts
4. Problem Formulation
5. Number Theory

6. The Integers
7. The Rational Numbers
8. Congruence
9. Equations and Inequalities
10. Decimal Representation for Rational Numbers
11. Probability
12. Measurement
13 Perpendiculars and Parallels (I)
14. Similarity
15. The Real Number System
16. Area, Volume, and Computation
17. Perpendiculars and Parallels (II)
18. Coordinate Geometry
19. Problem Solving
20. Solution Sets of Mathematical Sentences
21. Rigid Motions and Vectors
22. Computers and Programs
23. Quadratic Functions
24. Statistics
25. Systems of Sentences in Two Variables
26. Exponents and Logarithms
27. Logic
28. Applications of Probability and Statistics

The SSAM sequence consists of eight chapter bound in five units, followed by *Intermediat Mathematics, Part II.* The chapter titles are:

1. Organizing Geometric Knowledge
2. Concepts and Skills in Alegbra
3. Formal Geometry
4. Equations, Inequalities, and Radicals
5. Circles and Spheres
6. The Complex Number System
7. Equations of the First and Second Degree i Two Variables
8. Systems of Equations

*9. Logarithms and Exponents
10. Introduction to Trigonometry
11. The System of Vectors
12. Polar Form of Complex Numbers
13. Sequences and Series
*14. Permutations, Combinations, and the Binomial Theorem

* Optional chapters

Suggested Sequences and Grade Placements

A primary goal of this curriculum project is provide a flexible secondary school mathemati program. The typical plan is for average a above average mathematics students to begin SS in seventh grade. Students will vary in the leng of time they take to complete SSM. Those st dents who plan to take further mathematics w then take SSAM which provides a transition t variety of more advanced courses. Teachers w lect from *Intermediate Mathematics, Part II,* hose chapters which are needed for the remaining mathematics courses their students will take. hose classes which complete the SSM and SSAM quence by the middle of the junior year have a ariety of possibilities for the remaining three mesters.

Possible sequences of SMSG courses for teachers select from depending on the needs and interts of their students are:

Elementary Functions for one semester followed by *Calculus* for one year which would prepare students to take the BC Advanced Placement Exam in calculus;

For those who wish to take more mathematics but not calculus, one semester each of *Introduction to Matrix Algebra; Algorithms, Computation, and Mathematics*; and a course in Probability and Statistical Inference using one of the commercially available texts.

Calculus of Elementary Functions for one year, which would prepare students to take the AB Advanced Placement Exam in calculus, and one of the semester courses in option 2.

Classes that complete SSM and SSAM at the end of their junior year may choose:

Calculus of Elementary Functions for their last year, or

two of the three semester courses listed in option 2 above.

Slower mathematics students may study the M sequence for a longer period. Since SSM is signed to contain the mathematics which all izens should know, students who begin the seence should continue until they have committed it. It is of interest to note that in at least e high school, the lowest achievers in matheatics are beginning SSM as ninth graders and ll spend four years studying it.

esults of the Classroom Try-out

In 1967–68, 527 seventh grade average or above erage mathematics students from middle class cio-economic backgrounds began studying SSM. hey were in fifteen classes taught by fifteen different teachers in five schools in or near San Jose. f these students, 85% remained in the program rough the eighth grade, 58% through the ninth ade, and 37% through the tenth grade. In 70–71, there were eight tenth grade classes in ven high schools. These classes completed the M sequence in the fall and finished about half the SSAM sequence. Some schools have now ased those students who are continuing in

mathematics into their regular eleventh or twelft
grade mathematics classes, while others are co
tinuing with SMSG courses.

In 1968–69, another similar group of 448 se
enth graders taught by the same teachers bega
studying SSM. 86% of this group remained in t
program through the eighth grade, but only 19
were in it at the end of the ninth grade. Most
the students who dropped out after two yea
were entering high schools which had droppe
the program. This decision was made becau
there were too few SSM students attending the
high schools to make up full classes.

The try-out students were tested in the fall
their seventh grade year and in the spring of ea
year they remained in the program. Many of t
tests that were administered to SSM students h
also been administered to students in the five ye
National Longitudinal Study of Mathematic
Abilities (NLSMA) from 1962 to 1967. Statistic
comparisons were made between the SSM studen
and a sample of NLSMA students who had tak
a year each of algebra and geometry by the end
their sophomore year in high school. In the fall
seventh grade, the SSM students scored signi
cantly higher than the NLSMA students on t
intelligence scales that were administered,
understanding of and computation with who
numbers, and on understanding of geometric
relationships. The NLSMA students scored si
nificantly higher on computation with fractio
and decimals.

At the end of the ninth grade, the NLSM
students scored significantly higher than the SS
students on knowledge and understanding of alg
braic expressions and equations. This is to
expected since most of the work on algebr
manipulation occurs in SSAM (tenth grade
these students). By the end of tenth grade, SS
students scored higher on algebraic expressio
than NLSMA students who had taken this test
ninth grade but lower than those NLSMA st
dents who took it again at the end of eleven
grade after studying a second year of algeb
Similarly, when the SSM students were tested
the end of ninth grade on geometry, they scor
lower than NLSMA students given this test at t
end of tenth grade. But when the SSM studen
were given this test again at the end of ten
grade, they scored higher than the tenth gra
NLSMA students had. When both groups we
tested at the end of the same grade, SSM studen
scored higher than NLSMA students on algebr

equalities (ninth grade) and on measurement
enth grade). The two groups had the same
ean on a two-item scale on structure of proofs
lministered at the end of tenth grade.

The SSM students were also given some spe-
ially constructed tests designed to evaluate meth-
ls of developing topics that were used in certain
apters to see if these methods were effective.
esults of these tests were given to the writers to
se while revising the corresponding chapters.

The try-out teachers attended biweekly semi-
ars conducted by SMSG staff members. These
minars included discussion of the content of the
apters in the sequence and methods the class-
om coordinators had found useful in teaching
e pilot class. Time was also devoted to discuss-
g how the chapters currently being taught were
ing. Notes were made of the teachers' com-
ents. The teachers also turned in written evalua-
ons on each chapter. This information was used
the writers when the sequence was revised the
llowing summer.

At the end of the last seminar, the tenth grade
y-out teachers were asked to describe their stu-
nts. They were especially pleased with the per-
rmance of their students in reasoning, under-
anding verbal problems and setting up equa-
ons, understanding proofs, and in recalling and
ing previously developed concepts.

The coordinators of the classroom try-out phase
this project were William S. DeVenney and
mes C. McCaig. Their work included teaching
pilot class which studied the SSM chapters be-
re the try-out classes did, in order to warn the
y-out teachers of possible pitfalls. This class at-
nded Ortega Junior High School in Cupertino
nion School District and Fremont High School
Fremont Union High School District.

The following Northern California school dis-
icts, schools, and teachers participated in the
assroom try-out and data collection of SSM and
AM.

Campbell Union High School District
  Branham High School, San Jose
    Robert B. Canihan
  Leigh High School, San Jose
    Edwin D. Stone
Cupertino Union School District
  Joaquin Miller Junior High School, San Jose
    James D. Hughes
    James C. McCaig
    Kenneth B. McNeal

- Fremont Unified School District
  - Hopkins Junior High School, Fremont
    - Gordon W. Boike
    - Joan M. Cowgill
    - Delbert V. Sullivan
  - Mission San Jose High School, Fremont
    - George E. Crozier
    - George A. Tomlinson
- Fremont Union High School District
  - Lynbrook High School, San Jose
    - Stanley R. Boone
    - Otis T. Halliday
    - Robert L. Stone
- San Jose Unified School District
  - Bret Harte Junior High School, San Jose
    - Howard G. Esterline
    - Jane R. Pflughaupt
    - Robert P. Wilens
  - Leland High School, San Jose
    - Art DeVault
- Santa Clara Unified School District
  - Emil R. Buchser High School, Santa Clara
    - Ronald M. Hagelin
  - Santa Clara High School, Santa Clara
    - Kent G. Marshall
  - William A. Wilson Intermediate School, Santa Clara
    - Ralph R. Bessler
    - Robert J. Culbertson
    - Gary R. Leitz
- Union School District
  - Dartmouth School, San Jose
    - James C. Haugaard
    - Charles S. Hogdon
    - Clarence Kludt

In addition, several schools in the Los Angele
City School Districts used SSM for two years wit
accelerated students under the direction of Clyd
Corcoran. One school in San Diego County e
perimented with using SSM in various settings i
cluding a mathematics laboratory and small grou
instruction. This attempt was directed by Thoma
P. Davies. Feedback from these experiences wa
collected and distributed to members of the wri
ing team.

Implementation

The implementation of the SSM program i
volves a commitment of at least three years, b
ginning with Chapter 1 and starting generally a
the 7th grade. This is due to the fact that topi
usually taught in isolation are intermingled an
carefully sequenced. Therefore, it is imperativ

ıt the teachers involved in the program become
oroughly familiar with the entire program, espe-
lly the portion that precedes the part they are
rrently teaching.

The SSAM program is designed to provide a
nsition for the student from the SSM program
a more conventional mathematics program in
latter part of high school. SSAM has been
igned so that it may be flexibly implemented
pending upon the particular school situation.
achers may select from this program portions
ich will best facilitate a transition to whatever
ogram their school offers to students who have
cessfully completed SSM.

t is highly recommended that some in-service
ining program be provided for teachers who
l be teaching SSM. The Teacher's Commen-
ies that are provided for each of the chapters
rather extensive and should be helpful refer-
es in such a program. They contain notes re-
ding the role of certain class discussion exer-
s, the use of the spiral approach for certain
cepts, and the nature of any special develop-
nt used.

ormal courses for in-service teachers being
ined to teach SSM could concentrate on the
owing four areas which correspond to the cen-
themes of the SSM program. (The correspond-
SSM chapters are given in parentheses under
h theme.)

The Number Systems of Arithmetic and Alge-
bra

Stress would be placed on the structure of
the systems of non-negative integers, the inte-
gers, the rationals, and the reals as done in
SSM. Vincent Haag's *Structure of Elementary
Algebra*, Studies in Mathematics, Volume 3,
SMSG, could be used as a reference (SSM
Chapters 5–7, 10, and 15). Work on algebra
should be included as needed, with polynomi-
als in one variable over a given system possi-
bly serving as a final topic (SSM Chapters 9,
20, 23, 25, and 26).

Geometry

The SSM program contains a great deal of
geometry, including some coordinate geome-
try, rigid motions, right triangle trigonometry,
and vectors. The SSM geometry chapters could
be analyzed carefully using the SMSG text,
Geometry, as a reference and some attention
given to the role of logical inference, un-
defined concepts, definitions, postulates, and
theorems in geometry. Finally, the SMSG text,

Geometry With Coordinates, could be used
background for some coordinate geomet
(SSM Chapters 1, 8, 12–14, 16–18, 21, and 27

C. Probability and Statistics
This subject will be new to most teachers. T
three SSM chapters on this theme could
studied carefully using an introductory stat
tics and probability text as a reference (SS
Chapters 11, 24, and 28).

D. Problem Solving and Computer Mathematic
A great deal of attention is given to proble
solving and model building (SSM Chapters
4, and 19). Likewise, there is a substantial e
phasis given to the use of flow charts and co
puters (SSM Chapters 3 and 22). Therefo
these chapters should be analyzed carefully u
ing the references mentioned in the Teache
Commentaries as a source for supplementa
work.

Teachers who have had experience teachi
contemporary Plane Geometry and Algebra I
Trigonometry courses should need no in-servi
training for the SSAM program. The planni
time for implementing this program should
devoted to a familiarization with the content
the chapters and to a careful selection of tho
chapters most appropriate for a transition to t
mathematics courses offered in the final semest
of high school.

Description of a Low Achiever Version of SSM

In the summers of 1969, 1970, and 1971, thr
writers prepared a *Special Edition of Seconda*
School Mathematics for seventh and eighth gra
low achievers. This edition was based on findin
of low achiever studies by Sarah T. Herriot a
William S. DeVenney in SMSG Reports, Nos.
6, and 7. The chapters in this program introdu
the same general content areas as the first t
years' work in the version for average and abo
average mathematics students, but the mater
has been rewritten to appeal to students who
mathematics achievement in elementary scho
was very low. The quantity of reading and t
reading level have been reduced to a minimu
The student is provided with addition and mul
plication tables and with flow charts for compu
tional algorithms. Thus, he is relieved of as mu
computation as is possible and is free to conce
trate on mathematical concepts and relationshi
This program consists of the following chapt
which can be covered in approximately two yea

1. Flow Charts
2. Structuring Space

3. Functions
4. Number Theory
5. The Integers
6. Rational Numbers
7. Probability
8. Equations
9. Congruence
10. Decimals
11. Parallelism
12. Similarity
13. More About Rational Numbers
14. Perpendiculars
15. Measurement
16. Real Numbers
17. Solving Equations and Inequalities
18. Coordinate Geometry

Teacher's Commentary accompanies each set of ine chapters.

The material is presented to the student in the rm of work sheets; however, it is not, in the or- inary sense, a work book. Topics in each chapter e developed sequentially lesson by lesson. Teach- -led class discussion exercises are carefully pro- ammed to lead the student to successful expe- ences in the exercise sets. Each student should provided with a binder in which to place his mpleted lessons. This relieves the student of eping track of a textbook and allows him to ncentrate on what he has accomplished, rather an on what remains to be completed. Each apter concludes with (1) a practice test which sentially tells the student what he is expected know; (2) a chapter test which parallels the actice test; and (3) review exercises to promote tention of material covered in previous chapters.

As the cost of producing individual worksheets mmercially proved to be prohibitive, this ma- rial comes to the teacher in bound volumes. ch chapter consists of a number of lessons of e or more pages. Each page is perforated for sy removal. These pages can be reproduced in antities sufficient for the class by means of spirit asters. The cost of reproducing these materials the school is offset by the need to purchase only e set of student texts.

lassroom Try-out of Low Achiever Program

The preliminary version of *Secondary School athematics, Special Edition*, was tried out in 69–70 and 1970–71 in seventh and eighth grade asses of low achieving mathematics students lo- ted in San Francisco, Oakland, San Jose, and rthern suburbs of San Jose. The classes con- ted of students who were about two years below grade level in mathematics. Sixteen sevent grade classes with fourteen different teachers i eleven schools initially participated in the stud Eleven of these classes in ten schools continue in the study when their students were eight graders. Several classes consisted of disadvantage mixed urban minority (including Black) studen and two classes were primarily Mexican-America Most eighth grade classes completed sixteen of th chapters in the sequence.

During the two years of the study, the teache attended biweekly seminars which covered th content of the chapters (some of which was ne to most of the teachers) and the philosophy of th program. Problems which arose during the try-ou were discussed. Teacher evaluations served as basis for the final revision of these chapters th following summer. The reactions of the teac ers and students were extremely favorable. Th teachers commented particularly on how the st dents' attitudes toward mathematics and the self-concepts had improved during the two yea they were in this program. Many more studen than usual elected to enroll for more or high level mathematics as ninth graders than was r quired. Students who complete the *Special Ed tion* chapters are well prepared to study a cour such as SMSG's *Introduction to Algebra* over two-year period.

The following school districts, schools, and achers participated in the classroom try-out of is project.

- Cupertino Union School District
  - Joaquin Miller Junior High School, San Jose
    - Ann L. Bixby
  - Ortega Junior High School, Sunnyvale
    - L. Hart Andelin
- Moreland School District
  - Rogers Junior High School, San Jose
    - David J. Hallstrom
- Oakland Public Schools
  - Frick Junior High School
    - Katherine Maze
  - Hamilton Junior High School
    - Jimmie Neblett
- San Francisco Unified School District
  - Denman Junior High School
    - Marie J. Faraone
  - Everett Junior High School
    - Frances Leech
    - Stephen Malayter
  - Benjamin Franklin Junior High School
    - Robert Kane
  - A. P. Giannini Junior High School
    - Joseph L. Glikman
  - Herbert Hoover Junior High School
    - Katherine Buck
    - Jo Ann McNally
    - William Ota
- San Jose Unified School District
  - Peter Burnett Junior High School
    - Preston Luke

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SCHOOL
MATHEMATICS
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Newsletter No. 38

August 1972

Section I
An SMSG Statement on Objectives in Mathematics Education

Section II
Minimum Goals for Mathematics Education

SECTION I

AN SMSG STATEMENT ON OBJECTIVES IN MATHEMATICS EDUCATION

Introduction

Careful attention to objectives of mathematics education has, very recently and very suddenly, become much more important than was the case in the past. The reason for this is that dissatisfaction with the results of our educational system has become widespread and has led to demands for much more accountability on the part of school systems and teachers than in the past. Accountability, however, can only be meaningful when actual achievement is compared with hoped-for achievement, and to specify the achievements we hope for is to state the objectives of education.

One result of this concern has been pressure on schools and on teachers to state their educational objectives in enough detail so that the comparison can indeed be made. It is a fact that such an exercise is unfamiliar to most teachers and not easy to carry out, and much of what has been done recently is seriously deficient. Rather than dwell on the details of these deficiencies, we take this opportunity to indicate some important criteria that should be kept in mind when examining statements of educational objectives.

It must be admitted that SMSG has not indulged in the statement of educational objectives to any great extent, particularly in its curriculum development activities. When SMSG started work in 1958, a deliberate decision was made not to start by preparing detailed outlines of, and objectives for, the courses it was to prepare. The rather general outlines provided by the Commission on Mathematics of the CEEB, and the belief that understanding of mathematics was just as important as computational skill, were felt to be enough to point the way. And indeed, in each of the SMSG curriculum development projects, the same philosophy and a rather broad outline were all that were given to the writing teams. In view of the success of each of these projects, such guidance seems to have been enough.

On the other hand, when, in the early sixties, it became apparent that a careful, intensive evaluation of the various SMSG curricula was needed, the matter of the objectives of the various mathematics curricula then available became paramount.

The particular answer developed for the National Longitudinal Study of Mathematical Abilities was to specify objectives by means of sets of specific test items. These have all been published in Wilson (1968). Background information concerning the rationale and development of these tests can be found in Romberg and Wilson (1969).

In any case, we hope that the long preoccupation with educational objectives in the past not only of the NLSMA staff and consultants but also the SMSG Advisory Board* may have contributed usefully to the following discussion.

The original review and analysis of the literature was carried out in a seminar directed by Dr. E. G. Begle and attended by Professors Nicholas Branca and Decker Walker. The following graduate students of the Stanford University School of Education attended: Len Berk, George Eshiwani, William Geeslin, Joseph Green, Roger Jarvis, Helen McCullough, Bertrand Morin, Barbara Pence, Stanley Puryear, Sally Thomas, Norman Webb and Robert Wise.

Variation in Statements of Educational Objectives

Goals and objectives of mathematics education have been put forth by many individuals, ad hoc committees, and national organizations. We will not attempt to review, even sketchily, these many lists of objectives. Very useful reviews are to be found in the dissertations of Harding (1968) and Oakes (1965). The NCTM (1970) publication on the history of mathematics education in the U. S. is another useful reference.

A hasty look at even a few of these statements is enough to reveal that they have been phrased in a wide variety of styles. One result of this is that it is usually difficult to compare one list of objectives with another. A second result is that many statements of objectives are actually quite meaningless and hence useless.

However, a more careful look at, especially, the lists of objectives presented by the major committees over the years indicates that statements of objectives vary over a limited number of dimensions. A close look at each of these dimensions will be helpful.

1. *Content*

The first and most obvious dimension along which educational objectives can vary is that of the mathematical topic to which the objective refers. The topic might be arithmetic, or geometric, or a part of analysis, or logic, etc. Here are some examples.

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*A preliminary draft of this document was reviewed by the SMSG Advisory Board at its final meeting in April, 1972. The present version incorporates many suggestions and comments made by Board members.

> Mastery of the four fundamental operations with whole numbers and fractions, written in decimal notation and in the common notation used for fractions.
>
> Commission on Mathematics (1959), p. 19.
>
> Given a numeral in any base, two–twelve, the students will find the corresponding base ten numeral.
>
> Instructional Objectives Exchange (1968), p. 14, Grades 7–9.
>
> The ability to recognize, to name, and to sketch such common geometric figures as the rectangle, the square, the circle, the triangle, the rectangular solid, the sphere, the cylinder, and the cube.
>
> Joint Commission of the MAA and NCTM (1940), p. 54.
>
> The child should learn to estimate by the eye and to measure with some degree of accuracy of lengths of lines . . .
>
> NEA (1894), p. 110.
>
> Boys must learn to draw general figures; triangles must not be drawn isosceles, quadrilaterals should not be drawn as parallelograms.
>
> Mathematical Association (1959), p. 68.
>
> Introduction in grade 11 of fundamental trigonometry—centered on coordinates, vectors, and complex numbers.
>
> Commission on Mathematics (1959), p. iii.
>
> The student should understand and be able to correctly use the terms:
>
> simple statement, compound statement, negation, conjunction, disjunction, conditional, biconditional, logically equivalent, converse, contrapositive, direct proof, proof by contradiction, quantifier.
>
> SMSG (1971), Chapter 27, teacher's commentary, p. 2.
>
> Appreciation of mathematical structure ("patterns")—for example, properties of natural, rational, real, and complex numbers.
>
> Commission on Mathematics (1959), p. iii.

Although some of the variations may be attributed to grade level, nonetheless the above illustrations are enough to remind us that a wide variety of mathematical topics have been thought to be desirable for the school curriculum. Even so, they are not sufficient to demonstrate the very large number of topics which actually have been advocated at one time or another.

In addition to specific topics, such as those illustrated above, much broader descriptions of content are often made.

> Every phase of the mathematics curriculum, as well as all classroom procedures, would be scrutinized for the opportunities to develop a growing appreciation of the immense power of mathematics, of its record as a universal servant of mankind, of its cultural significance, and of its permanent place in the study of nature, in science, in the practical arts, in business, engineering, and everyday life.
>
> Joint Commission of MAA and NCTM (1940), pp. 68, 69.

> . . . pupils will have gained an actually real insight into the true nature of mathematics and an enthusiasm for it.
>
> Joint Commission of MAA and NCTM (1940), p. 74.

> To see mathematics as a human endeavor which demands creative energy.
>
> Beberman (1958), p. 38.

> Understanding of the nature and role of deductive reasoning in algebra, as well as geometry.
>
> Commission on Mathematics (1959), p. 33.

2. *Cognitive level*

When a student adds two three-digit numbers he is working at a different cognitive level (or is using a different cognitive process) than when he selects from a set of geometric shapes those which are not convex. When he locates the maximum values, over an interval, of a particular polynomial function, he is working at still another level.

There are many schemes for the categorization of cognitive levels. However, most of them have evolved from the taxonomy defined by Bloom (1956). The categorization scheme which will be used in this report is essentially the one used in the NLSMA study. This scheme also evolved from the Bloom Taxonomy and, as in the taxonomy, each category is more complex than the previous one. In some cases, the categories can even be considered as hierarchical. That is, a lower category may be a necessary prerequisite for a higher category. The five categories which we find most appropriate for precollege mathematics education are *Knowledge of Facts, Computation, Comprehension, Application,* and *Analysis*. A more precise definition of each cognitive level category is as follows.

Knowledge of Facts—Objectives which require the recall of specifics such as terminology, symbols, or conventions; emphasis is on recall, not upon synthesis, generalization, or translation of the recalled information.

Computation[*]—Objectives which require straightforward manipulation of problem elements according to rules the subjects presumably have learned; emphasis is upon performing operations, not upon deciding which operations are appropriate.

Comprehension—Objectives which require either recall of concepts and generalizations or transformation of problem elements from one mode to another; emphasis is upon demonstrating understanding of concepts and their relationships, not upon using concepts to produce a solution.

Application—Objectives which require (1) recall of relevant knowledge, (2) selection of appropriate operations, and (3) performance of the operation. Objectives are of a routine nature requiring the subject to use concepts in a specific context and in a way he has presumably practiced.

Analysis—Objectives which require a non-routine application of concepts. Under this heading come problem formulation and mathematical modeling.

Categorization of objectives by cognitive levels is complicated by other factors. For example, an objective which is classified as computation for one age population may be classified as application or even analysis for younger populations.

An example of an objective which can be categorized as *Knowledge of Facts* is:

> Ability to know and name correctly the sphere, cube and cylinder and the most characteristic surface forms such as circles.
>
> International Commission (1911), p. 286.

An example of the *Computation* category is:

> To perform the fundamental operations with fractions.
>
> Breslich (1930), p. 205;

of the third category of *Comprehension* is:

> . . . the student should understand that the

---

[*] Description of the last four categories are taken from Romberg and Wilson (1969), pp. 39–40.

> solution set of a linear inequality such as $Ax + By + C > D$ consists of the points in a half plane.
>
> SMSG (1971), Chapter 25, p. 2.

An objective of the *Application* category is:

> He (the student) is expected, among other things, to solve equations (singly and in systems), ...
>
> Beberman (1958), p. 3.

Analysis, the final category, is illustrated in the following example.

> He (the student) should begin to devise constructions and demonstrations for himself.
>
> NEA (1894), p. 115.

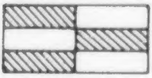
Many objectives include a combination of the different categories. For example, consider this last objective which involves both Computation and Comprehension:

> The development of an understanding of the deductive method as a way of thinking and a reasonable skill in applying this method to mathematical situations.
>
> Commission on Mathematics (1959), p. 23.

It is relatively easy to construct tests which measure student achievement at the cognitive levels of Facts and Computation. However, it does not seem to be widely appreciated that it is also possible to measure student achievement at the higher cognitive levels. For a full discussion of this, see Chapter 3 of Romberg and Wilson (1969). Here, however, are two sample test items, suitable for junior high school students, to illustrate each of the last three cognitive levels.

Comprehension

1. The product of 356 and 7 is equal to
   (A) $(300 \times 7) + (50 \times 7) + (6 \times 7)$
   (B) $356 + 7$
   (C) $(300 + 50) + (6 \times 7)$
   (D) $(3 \times 7) + (5 \times 7) + (6 \times 7)$
   (E) $300 \times 50 \times 6 \times 7$
2. This drawing suggests a rational number. Choose the fraction on the right which names the same rational number.



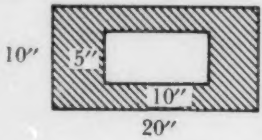
(A) $\frac{1}{2}$
(B) $\frac{2}{3}$
(C) $\frac{1}{3}$
(D) $\frac{3}{4}$
(E) None of these

Application

1. A family spent an average of \$46 per month on food for a 2-month period. If food cost \$39

one of those months, how much was spent on food the other month?

(A) $32
(B) $42.50
(C) $49.50
(D) $53
(E) $60



2. What is the *area* of the shaded portion in the figure shown above?
   (A) 25 square inches
   (B) 30 square inches
   (C) 50 square inches
   (D) 100 square inches
   (E) 150 square inches

Analysis

1. A chess club ran a weekly tournament in which every member played every other member just once. When *one more* member was admitted, it was found necessary to play *eight more* games per tournament. Now how many members are there in the club?
   (A) 20
   (B) 16
   (C) 12
   (D) 9
   (E) 8
2. In a certain classroom, the teacher notes that when the attendance is 92% there are exactly seven empty seats, but when the attendance is 88% there are eight empty seats. What is the total number of empty seats when the attendance is 100%?
   (A) five
   (B) four
   (C) three
   (D) two
   (E) none

3. *The Affective Dimension*

Many of the objectives that have been stated for mathematics education refer to attitudes or feelings about mathematics. Although a taxonomy of affective educational objectives has been published, Krathwohl (1964), it has received very little attention. In particular, very little use of this taxonomy has been made in the design of tests or questionnaires or in the design of research studies. Consequently, this taxonomy has had very little empirical validation.

In addition, while there has been much interest in students' attitudes towards mathematics, as indicated in the recent review by Aiken (1969), practically nothing seems to be known about pedagogical procedures which can be used to modify these attitudes. While we are in agreement tha we would like students to end up with a favorable attitude towards mathematics, we do not know what to do to improve student attitudes. Consequently, only enough examples of statements o affective objectives are given below to illustrate the wide variety that can be found among them. (Note that additional illustrations appeared in the previous two sections.)

> To convey an appreciation of the use of formulas, graphs, and simple equation.
> Harvard Committee (1945), p. 162.

> The building of confidence in one's own analytical powers.
> Cambridge Conference (1963), p. 9.

> Satisfaction with thorough work and precision of statements.
> Smith and Reeves (1927), p. 183.

> Enjoyment of mathematics.
> National Assessment of Educational Progress (1970), p. 30.

> To feel that this activity (processes in mathematics) is worthy of earnest pursuit.
> Beberman (1958), p. 37.

> An understanding of mathematics as a continuing creative endeavor with aesthetic values similar to those found in art and music. In particular, it would be made clear that mathematics is a living subject, not one that has long since been embalmed in textbooks.
> Commission on Mathematics (1959), p. 11.

> A degree of interest in mathematics which will encourage the pupil to continue in the study.
> Breslich (1930), p. 207.

> To give the student an appreciation of postulational thinking.
> Pre-Induction Courses in Mathematics (1943), p. 123.

> Appreciation of the contributions mathematics has made to the progress of civilization.
> Breslich (1930), p. 206.

4. *Verifiability*

This dimension refers to the degree to which it is possible to ascertain whether or not a particular student has achieved a particular objective. Actually, this dimension can be resolved into (at least) three separate subdimensions. Progress oward an objective may be difficult, or impossible, o verify because

1) the statement of the objective is ambiguous or lacks specificity;
2) we do not know how to measure progress toward the objective;
3) there are technical problems which make it difficult to measure, with precision, progress toward the objective.

First, let us give an example of an objective vhich presents no difficulties along any of these ubdimensions.

The ability to express in decimal and percent orm the fractions:

½ ⅔ ¾ ⅜ ⅞ ⅓ ¼ ⅛
⅝ ⅙

Smith and Reeves (1927), p. 191.

It is clear that an objective, to be verifiable, nust be stated sufficiently clearly and specifically so that any two qualified evaluators can agree on he universe of test items that would be valid for ests to measure student progress toward the obective.

Variation along this subdimension actually appeared in the very first two illustrations in section 1, and a review of the various illustrations in the preceding sections demonstrates that the degree of clarity and specificity with which objectives can be stated is quite large.

Here are some more statements of objectives which are not clear and specific.

> A familiarity with the basic concepts, the processes, and the vocabulary of arithmetic.
>
> Joint Commission of MAA and NCTM (1940), p. 54.

> The objectives of a course in plane geometry should be to acquaint the pupil with geometric facts and their applications and to give him an appreciation of postulational thinking.
>
> Pre-Induction Courses in Mathematics (1943), p. 123.

> We wish pupils to develop the conscious use of a technique of thinking.
>
> Beatley (1935), p. 331.

> An understanding of the general properties of geometrical figures and the relationships among them.
>
> Commission on Mathematics (1959), p. 11.

> . . . the task of "solving novel problems and puzzles."

National Assessment of Educational Progres
(1970), p. 28.

As for the second subdimension, we have alread pointed out that we are very poorly equipped t measure affective variables and hence to find ou how far a student has progressed toward a particular affective objective. But even outside th affective domain we have measurement problems Look back at the last three illustrative objective in section 1. How do we find out how far a student has progressed toward any one of these? Similarly, examine for verifiability the last illustrativ objective in section 2.

As to the last subdimension under this heading, refer again to the first illustrative example in this section. Ten fractions are listed and the objective is to be able to express each of them in decimal and in percent form, a total of twenty separate tasks. It is easy to see how to find ou how far a student has progressed toward this objective: give him these twenty tasks, and observe how many he can do correctly.

But now let us consider this objective:

> Learning column addition to six one-digit addends and four three- and four-digit addends.
>
> Flournoy (1964), p. 10.

There are more than six trillion addition problems which fall into the category specified in this objective. Obviously, we do not ask a student to work each of these problems. Instead, we choose a random sample of such problems to constitute a test. A student's score on such a test gives us an estimate of his achievement with respect to the objective. The statistical theory of such estimates is well known.

Next, consider each of these two objectives:

> The scholar should be thoroughly trained in performing correctly and rapidly the four fundamental operations with integers, vulgar fractions and decimals.
>
> NEA (1894), p. 105.
>
> Ability to solve linear equations containing common or decimal fractions.
>
> Smith and Reeves (1927), p. 202.

In either case the number of problems which a student is expected to be able to solve is infinite. There is no unique procedure for drawing a finite random sample from an infinite universe, and therefore there is no well defined way of estimating a student's achievement with respect to either of these objectives.

Usually we avoid this difficulty by tacitly limiting the physical size of allowable test items, thus making the universe of test items finite.

The same problem arises with objectives at higher cognitive levels. Again, tacit limitations on the nature of test items are employed. Though it is seldom done, it would be helpful if these limitations were made explicit.

5. *Feasibility*

This dimension refers to the extent to which we, as teachers, are able to move students toward a particular objective. Refer again to the first illustrative objective of the preceding section. We know by experience that we can teach most students, at the junior high school level, to transform fractions into percents or decimals. Consequently, this objective is feasible for most students.

On the other hand, consider these objectives:

> The student should develop study skills which will enable him to study mathematics independently and effectively.
>
> Harding (1968), p. 101.
>
> Ideal of perfection as to logical structure, precision of statement and of thought, logical reasoning (as exemplified in the geometric demonstration).
>
> Oakes (1965), p. 36.
>
> To discover new geometrical facts.
>
> Breslich (1930), p. 205.
>
> A gradual and increasing development of the ability to manifest coherent, logical thinking in everyday life situations.
>
> Joint Commission of MAA and NCTM (1940), p. 67.
>
> The development of the habit and ideal of persistence.
>
> Breslich (1930), p. 208.

It is not at all clear that we know how to teach our students to improve their performance with respect to any of these objectives. Also, we have already noted that none of the affective objectives which have appeared in the literature have been demonstrated to be feasible.

6. *Population*

The dimension of population identifies the individual or group of individuals for which the objective is stated. The variation within population may be with respect to not only age level but also social-economic status, mental maturity, future aspirations, mathematical background, regional location, etc.

In general, a set of objectives is intended for a particular population. For example, the NEA (1894) Report was primarily for secondary education but several objectives or recommendations were given for grammar school (6–13 years of age). The International Commission (1911) gave a few precise objectives for kindergarten mathematics but was mostly concerned with the elementary schools. The UICSM's curriculum and objectives (Beberman, 1958) stated explicitly that their program was intended for the college-bound.

Population is often stipulated within the objective itself. For example,

> Intelligent guidance should guarantee that every high school pupil study mathematics according to his need and ability.
>
> Pre-Induction Courses in Mathematics (1943), p. 185.

Here the population addressed contains only one, the individual.

To the other extreme, many objectives are written for a large population. For example,

> Every high school pupil must be able to compute with assurance and skill. . . .
>
> Pre-Induction Courses in Mathematics (1943), p. 115.

Between these two extremes lie several different subdivisions.

> . . . for the *mathematically less gifted pupils* in the ninth grade there is little straightforward mathematics available beyond elementary instruction in arithmetic and informal geometry, which should include guidance in the use of formulas, equations, graphs, and right triangles.
>
> Harvard Committee (1945), p. 163.

> For some, *those with little or no mathematical background*, this arithmetic plus certain essential topics from general mathematics such as scale drawing, including elements of blueprint reading, numerical trigonometry, informal geometry, and simple formulas and equations, will constitute the course.
>
> Pre-Induction Courses in Mathematics (1943), p. 121.

The last two examples restrict the population by ability in mathematics. The former concerns innate ability, whereas the latter is more concerned with previous mathematics courses taken by the students.

One last example illustrates how some objectives are restricted in population by the future aspirations of the student.

> The prospective candidate for admission to college certainly needs instruction in both algebra and demonstrative geometry.
>
> Harvard Committee (1945), p. 167.

These examples only illustrate a few of the ways in which population can be designated or subdivided. What is important is that the designation and subdivision is very important to the meaning of the objective.

7. Form

This dimension has three values, Behavioral, Expressive, and Pedagogical.

Behavioralists believe that intended educational changes should be expressed in terms of measurable student behaviors. W. James Popham describes a behavioral objective as follows:

> A satisfactory instructional objective must describe an observable behavior of the learner or a product which is a consequence of learner behavior.
>
> Popham (1969), p. 35.

As an example of a behavioral objective, consider the following objective:

> Given a whole number between one and one hundred, the student will identify it as a prime or a composite number.
>
> Instructional Objectives Exchange, grades 4–6 (1968), p. 58.

There is some disagreement on the amount of specificity required. Some believe that the important outcomes of education are rather global and inarticulate attitudes, that concepts are developed *gradually* as a by-product of lessons, and that the important changes can rarely be observed over short time-spans. Those holding this view might select this objective:

> The major role of mathematics in developing desirable characteristics lies in the contributions it can make to growth in the ability involved in reflective thinking or problem solving.
>
> Progressive Education Association (1938), p. 59.

However, while it is harder to construct a good test of problem solving ability than it is to develop a test on prime and composite numbers, the second of these two objectives is no less behavioral than the first.

Eisner, a major proponent of expressive objectives, describes this type of objective as follows:

> An expressive objective describes an educational encounter; it identifies a situation in which children are to work, a problem with which they are to cope, a task in which they are to engage; but it does not specify what from that encounter, situation, problem, or task they are to learn.
>
> Eisner (1969), p. 15.

An example of an expressive objective in mathematics is:

> To expose the student to these ideas (concepts of problem formulation and mathematical modeling) as broadening experiences. . . .
>
> SMSG (1971), Chapter 4, teacher's commentary, p. 48.

Pedagogical objectives state how something should be taught. Usually it is clear what the student should learn, but empirical evidence that the method really works is very seldom provided. Here are some examples:

> Mathematics is by no means the only road to an appreciation of abstraction and logical structures. But tactically these ends may be approached most readily through mathematics, particularly with the young.
>
> Harvard Committee (1945), p. 162.

> A very large number of teachers would be well advised to tackle a greater variety of constructions than they are accustomed to, and to lay more stress on them.
>
> Mathematical Association (1959), p. 75.

Comments on the Dimensions

A. It is clear that while these dimensions can be delineated separately, they must be considered together in practice. We have already noted an interaction between population and cognitive level. Addition of two three-digit numbers is a computational task for a sixth grader but requires analysis of a second grader. Population also interacts with content.

The goals:

> . . . Fundamental theorem of algebra, winding number, location of roots.
>
> Cambridge Conference (1963), p. 44,

even today would not be unreasonable for selected high school seniors. However, despite Bruner's famous statement (". . . any subject can be taught effectively in some intellectually honest form to any child at any stage of development." Bruner [1960], p. 33), it would be nonsense to advocate this goal for first graders.

Feasibility probably interacts also with population. Abstract deductive reasoning, for example, does not seem to be a feasible objective for elementary school students, but it is for many older students.

And, of course, there is no point in talking about the feasibility of an objective if we know it cannot be verified, or even if we don't know that it can be verified.

B. The advocator of any specific objective for mathematics education should expect to be asked "why?". The answer to such a question begins with the word "because" but can continue in two different ways. One way would be along the lines of: "achievement of this objective is a good thing in itself." The other would be along the lines of: "it is a prerequisite to learning another topic."

Note that the first kind of answer expresses a value judgment while the second makes a factual statement. Now, when two individuals differ with respect to a particular value statement, there is no rational way of adjudicating the matter and we can only note that they differ. However, differences with respect to purportedly factual statements can be examined empirically.

Thus it appears that the statement of an objective is incomplete unless it is accompanied by the statement of its purpose, the answer as to why the objective was selected.

Many of the sets of objectives that have been published, and this is particularly true of some of the sets of behavioral objectives which have appeared recently, are seriously deficient because they lack statements of purposes.* Inspection of the individual objectives in these sets rarely yields one which is clearly intrinsically important, and for many others it is hard to find important objectives for which they are prerequisite.

An SMSG Statement on Objectives in Mathematics Education

As mentioned in the introduction, the importance of the objectives of mathematics education has recently been substantially enhanced. The

*A second objection to many of the presently available sets of behavioral objectives is that the cognitive levels of knowledge and computation are heavily overrepresented, while the levels of comprehension, application, and analysis are badly neglected.

SMSG Advisory Board hopes that those stating objectives and also those who use statements of objectives will keep the following principles in mind.

I. Statements of objectives should be hortatory. They should be taken seriously by teachers, curriculum workers, and textbook writers as important and realistic guidelines. They should *not* be expressions of wishful thinking.

II. On the other hand, statements of objectives should be taken as floors, not ceilings. If a teacher or a school can go beyond stated objectives, so much the better.

III. If the statement of a particular objective is to be taken seriously, then the purpose of the objective has to be made clear. Further, a serious, relevant objective must be so clearly characterized as such as to be easily distinguishable from a personal whim.

IV. If statements objectives are to be taken seriously, then the objectives must be clearly verifiable and feasible. It is not enough to know that an objective has not been shown to be infeasible. Before it should be advocated, it should have been positively shown to be feasible (and verifiable).

V. To be consonant with the above, we believe that all statements of mathematics educational objectives should be put in terms of student behavior. [The one exception is that we advocate a particular pedagogical objective:

Teach understanding of a mathematical process before developing skill in the process.

We believe there is enough empirical evidence in favor of this to make it a realistic objective.]

VI. Also, to be in conformance with point IV, we advocate at present no affective objectives. There is no evidence available to show that attitudes toward mathematics can be manipulated, so such objectives are not, at present, feasible.

VII. None of the above should be taken as suggesting that we ignore goals which are, at the moment, not feasible or not verifiable. Indeed, such goals indicate the most important areas in which to concentrate our future research efforts.

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SECTION II

MINIMUM GOALS FOR MATHEMATICS EDUCATION

The School Mathematics Study Group recently completed a new mathematics curriculum, *Secondary School Mathematics,* for junior high school students. A full description of this curriculum can be found in SMSG Newsletter No. 36.

At its final meeting, in April 1972, the SMSG Advisory Board reviewed and approved this new curriculum and decided that one aspect of it was so important that it should be specifically called to the attention of the mathematical community in an SMSG Newsletter.

Secondary School Mathematics was designed to include those mathematical concepts, and *only* those, which all citizens should know in order to function effectively in our society. A list of these concepts is therefore SMSG's statement of the minimal goals of mathematics education. This section of this Newsletter provides a brief summary of these minimal goals.

The writing team which produced this new curriculum was asked to provide, for each chapter, reasons for including the chapter, or, in other words, an explanation of how the concepts in the chapter could contribute to effective functioning in our society. In response, the writers provided, for each chapter, one or more paragraphs listing the objectives of the chapter and others stating the purpose of the chapter.

These paragraphs, reproduced in what follows, provide our brief summary of the minimal goals of mathematics. Before turning to them, however, some comments are necessary.

While the writers used the word "objectives," it was not at all in the sense in which the word is used in Section I of this Newsletter. The analysis of objectives in Section I was carried out after the writers had finished their work. From the point of view of that analysis, the writers' "objectives" could well be replaced by "goals."

Nevertheless, some aspects of the analysis can be applied to the writers' statements.

It has already been mentioned how the content of *Secondary School Mathematics* was selected. The writers' statements, however, are much too brief to be able to specify in any detail the emphasis and depth of coverage given to any particular concept. Such information can only be obtained from inspection of the actual text materials.

At the end of the teacher's commentary for each chapter is a set of illustrative test items. Inspection of these will show that the writers expected students to be able to treat most mathematical concepts in the chapter at all cognitive levels. These test items also demonstrate that the writers' goals are verifiable.

It is interesting to note that the writers did not list affective subjectives.

As reported in Newsletter No. 36, *Secondary School Mathematics* underwent substantial pilot testing and this curriculum is unquestionably feasible, in its regular version for average and above average students, and in its special edition for less able students.

Some of the writers' objectives are stated in terms of student behaviors, but a considerable amount of translation of the writers' objectives into behavioral form would be needed before any empirical evaluation either of the curriculum or of teachers using it could be done.

Finally, when the writers' statements were reviewed by the Advisory Board, some comments on specific chapters were made and are included in what follows. A few more general comments are placed at the end.

SUMMARY STATEMENTS OF THE GOALS OF *SECONDARY SCHOOL MATHEMATICS*

Chapter 1–STRUCTURING SPACE

Purpose

This introductory chapter contains several fundamental ideas that are essential for the further study of mathematics. Specifically, the groundwork is laid for later chapters on geometry. The idea of the coordinate line and coordinate plane will be encountered throughout the course. In addition, the "structing of space" provides a basic example of abstracting a mathematical model from the physical world. The importance of this latter idea is a continuing theme.

Objectives

Sections 1 through 8: The student should understand that geometry is an *ideal* model of certain aspects of the physical world; a full appreciation of this is quite difficult. By explicit definitions and characterization by properties, we try to assure that various people's ideas (of plane, for instance) are in essential agreement. These sections are largely involved with definitions; the student should know them and be able to apply them at least to the analysis of a given figure an to making a correct drawing of a figure havin given properties.

Sections 9 through 12: The important point i that by the coordinatization, every point i named by a number pair and every number pai names a point. The ability to name given point by their coordinates and to locate the point having given coordinates is developed through extensive exercises and is the essential “skill.”

Comments

Additional objectives to be included:

Knowledge of geometric facts.

Applying these geometric facts to “real world” problems.

Learn to visualize 3-dimensional representation learn to communicate ideas with regard to spac and be able to interpret geometric ideas with physical world.

It is suggested that some statement might b made to the effect that problem-solving still enter in everyone's needs and that problems requirin geometric sense (measurement problems, coordinate systems, angles, etc.) are in abundance in application.

Chapter 2—FUNCTIONS

Purpose

The function concept is certainly one of the most significant ideas in mathematics. This first exposure to functions is intended to be gentle. There are many examples of functions and of various ways that functions may be represented. The student is not expected to master the concept. As he progresses through the sequence the function idea will recur in almost every chapter. Since physical situations often lead to functions and graphs, we pursue the idea of mathematical modeling. The use of the terms input and output of a function serves as an introduction to Chapter 3.

Objectives

The student should be familiar with the notation, language, and various means of representing functions. In particular, he should understand that with each input corresponds exactly one output.

The student should be able to recognize simple functions in various forms. For reasonably simple functions, he should be able to find the outpu given the input.

Comment

Objectives, second paragraph, first sentence: Students should be able to recognize simple functions in various forms such as, tables, equations, diagrams, statements and graphs.

Chapter 3—INFORMAL ALGORITHMS AND FLOW CHARTS

Purpose

Algorithms are important in almost every branch of mathematics. In developing an algorithm we must construct a detailed step-by-step plan. Flow charting offers excellent insight into the nature of algorithms and provides an introduction to a later chapter on computers and programming. Even divorced from its use with computers, the material on flow charts is valuable in itself. Flow charts are included wherever feasible throughout the course. The particular example of the division algorithm for integers is exploited in Chapter 5 (Number Theory).

Objectives

The student should gain some familiarity with the concept and use of flow charts and functions of various boxes in a flow chart. He should understand that the processes used to find solutions of problems are often step-by-step procedures that we call algorithms. These procedures are particularly useful in work with computers. The student should develop the skills necessary to express simple mathematical algorithms in flow chart form.

Chapter 4—PROBLEM FORMULATION

Purpose

The primary goal of the chapter is to examine a bit more closely the topic of mathematical modeling. At the heart of applications of mathematics is the process of finding an appropriate mathematical model of a real world situation. In fact this is often extremely difficult. We strive, in this chapter, to stimulate the student to realize that real world problems do not always resemble textbook problems. Chapter 4 is exploratory. No *specific* mathematical ideas are developed. Problem *formulation* is stressed here, not problem solving.

Objectives

We intend this chapter to be a soft-sell introduction to the concepts of problem formulation and mathematical modeling. We want the student to be exposed to these ideas as broadening experiences and for this reason have not made demands on his competency. In presentation of this chapter, we encourage an informal approach with a lot of student participation in open-ended discussions. The student may find this format unfamiliar and may need extensive guidance at the start with regard to how to approach the task of problem formulation. We include possible directions along which he might be steered, but we hope that this "guidance" would not so structure his thinking that he finds it necessary to fit his thinking into a mold. In other words, we appreciate that the student has imagination and do not wish to stifle it. For this reason, the "solutions" to the exercises consist only of suggested solutions and should not be considered to be irrefutable.

In order to tip the scale toward attaining these objectives, we provide more illustrative examples and exercises than one might use for a week's discussions. We hope that by doing so, the teacher can *pick and choose* examples that are likely to be better-received according to the interests of the particular class.

Our aim is to encourage open-endedness and creativity as well as an enthusiastic attitude toward the ideas of problem formulation, ideas which in all likelihood, heretofore, are unfamiliar to the student. We have included no suggested test items for this chapter. In fact, to promote the acquisition of this attitude toward problems, we advocate a relaxed classroom atmosphere and we further advocate that NO TEST be administered on this chapter!

Comments

Students are given a situation and then they are to answer the question, "What are some mathematical problems coming out of the situation?"

Chapter 5—NUMBER THEORY

Purpose

Students have had a good deal of experience with whole numbers. This chapter reviews some familiar concepts and develops some extremely important ideas including the statement of the Fundamental Theorem of Arithmetic. Other concepts considered are those of the greatest common divisor and the least common multiple. Although these number theoretic results are interesting in themselves, the primary motive in presenting them is because they are necessary tools in the treatment of rational numbers, real numbers, polynomials, and factoring. The chapter contains the first proof; includes proof that there is no largest prime. The square theorem will be needed in Chapter 15. A summary of the whole number properties is included. The structure of the system of integers follows in Chapter 6.

Objectives

By the end of the chapter, the student should understand the division algorithm and the concept of divisibility. He should know the meaning of primes, how to find some primes, and how to write a composite number as a product of primes.

Comment

We saw no test items for *understanding* the division algorithm.

Chapter 6–THE INTEGERS

Purpose

This chapter is really preliminary to Chapter 7. Rather than combine the problems of negative numbers with the manipulations of rationals, the negative integers are introduced here. The extension of the number line, the order relation, addition, subtraction, and multiplication using negative numbers are illustrated using negative integers. The important structural difference between the system of whole numbers and the system of integers may be summarized by the statement: "If a and b are any integers, then there is exactly one integer x such that $a + x = b$." The comparable statement for whole numbers is false.

Objectives

The system of integers, in itself, is not particularly interesting nor important for the student. Our goal is to separate the problems of manipulating "negative" numbers from the problems of manipulating rational numbers (Chapter 7). Once he student has learned, for example, that

$$(-2)\ (9) = -18$$

e can handle

$$\left(-\frac{2}{3}\right)\left(\frac{9}{4}\right)$$

first writing

$$\left(-\frac{2}{3}\right)\left(\frac{9}{4}\right) = -\left(\frac{2}{3}\right)\left(\frac{9}{4}\right)$$

d then he can concentrate on the problem of ltiplying the *positive* rationals $\frac{2}{3}$ and $\frac{9}{4}$.

Comments

At end of *objectives* paragraph, add: "Given any two integers, the student can find the sum, difference, product, and, if it exists, the quotient."

The board disagrees with the first sentence in the *Objectives* paragraph.

The topic of absolute value provides a new way of illustrating the concept of function.

Chapter 7–THE RATIONAL NUMBERS

Purpose

It is important that the student acquire skill in handling rational numbers. In this chapter the techniques for arithmetic operations with rational numbers are developed. Further, we wish to lay the foundations for later chapters. In Chapter 9 we shall solve equations of the form $ax + b = 0$ and rational numbers are needed. In Chapter 15, the student learns that although the rational numbers are dense they are not sufficient to "fill up" the number line nor to solve such equations as $x^2 - 2 = 0$.

Objectives

A first objective is to review the ordinary arithmetic of the nonnegative rationals. The usual rules for addition and multiplication are explained. Secondly, we wish the student to carry over his knowledge of manipulation of integers (positive, negative, and zero) in order to apply them to the rational numbers. Finally, we wish the student to recognize two fundamental differences betwen the structure of the rational numbers and the structure of the integers. Most important is the existence of the multiplicative inverse (reciprocal) of a non-zero rational. In addition, the rational numbers are dense; this property is not shared by the integers.

Comment

Add to *objectives:* to recall skills with nonnegative rationals and to extend them to all rationals.

Chapter 8–CONGRUENCE

Purpose

The notion of congruence (of plane figures) is necessary in order for us to pursue the study of geometry which was begun in Chapter 1 and is continued in several of the later chapters. Congruence is also used as a basis for measurement (Chapter 12). Apart from its relation to other chapters, Chapter 8 is important for other reasons. The student is given some experience with proofs.

Further, there are many practical applications of congruence.

Objectives

It is anticipated that the student will develop an understanding of the general nature of congruence. In particular, the student should be able to recognize when SAS, ASA, SSS situations occur for triangles and be able to draw correct conclusions.

Much of the geometric vocabulary introduced here is new. It will be reviewed and reinforced in subsequent chapters. For example, the subject of perpendicularity is taken up in more detail in Chapter 13. The student should be able to *use* the vocabulary. He should recognize, say, a quadrilateral, without necessarily being able to give a formal definition.

This is the student's first exposure to geometric proof. Obviously complete mastery of the nature of proof is not expected. It is sufficient that the student understand the proofs of the isosceles triangle theorems. More proofs will be encountered later.

Comments

The last sentence of the first paragraph, "Further there are many practical applications of congruence." really should lead the paragraph.

The kind of proof referred to in the statement should be described. Perhaps, ". . . using logical reasoning to establish facts (relationships) in geometry, . . ."

Change "geometric proof" to "geometric reasoning" in first sentence of last paragraph.

Chapter 9—EQUATIONS AND INEQUALITIES

Purpose

This chapter presents a systematic development of methods for solving some simple first degree equations and inequalities in one variable. It is not intended here to develop complete mastery of the process of finding solution sets of equations and inequalities. This topic will be spiraled consistently throughout the remainder of the curriculum. This chapter also provides an analysis of how these equations and inequalities may be formulated as models of problem situations.

Objectives

The central objectives of this chapter are

(1) to translate English phrases into mathematical expressions and to develop equa-

tions as models of certain problem situations,

(2) to solve equations of the form $ax + b = c$ and $ax + b = cx + d$ using the addition and multiplication properties of equality and the field properties of the rationals,

(3) to use a graphical approach in solving these equations,

(4) to develop inequalities as models of certain problem situations,

(5) to solve inequalities in a way similar to solving equations,

(6) to review properties of integers and rationals and to increase computational skill in their use.

Chapter 10—DECIMAL REPRESENTATION FOR RATIONAL NUMBERS

Purpose

In this chapter we revisit the rational numbers. In Chapter 7 (The Rational Numbers), the emphasis was on extending the number system and on the operations with rational numbers expressed as fractions. Here we consider the rational numbers as ratios, as percents, and as repeating (or terminating) decimals. Rounding off and computing with rational numbers expressed as decimals is reviewed and explained in the last section. These topics will be used frequently in later chapters, including Chapter 11 (Probability) and Chapter 12 (Measurement). An example is given of an infinite decimal that does not represent a rational number. This idea will be extended in Chapter 15 (The Real Numbers).

Objectives

After studying this chapter, students should be able to express a ratio as a rational number, express simple fractions as percents, write a decimal in expanded notation, find the decimal representation of a rational number and the simplest fractional name of a terminating or repeating decimal, and round off and compute with decimals. He should know that there are infinite decimals which are not rational numbers.

Comment

A realization that ratios are useful should be included among the objectives.

Chapter 11—PROBABILITY

Purpose

Some understanding of probability and statistics is essential for the educated citizen in modern society. This chapter introduces some fundamental ideas about probability. Probability theory is a requisite for the techniques of statistical analysis and statistical inference that play so large a role in industry, government, economics, social science and all branches of physical and biological science. In Chapter 24, some elementary statistical analysis is presented. Some basic tools of statistical inference are developed in Chapter 28, using the ideas of Chapters 11 and 24.

Objectives

We hope to leave with the student an awareness that many situations or experiments lead to results that are uncertain. In this introduction to probability we attempt to quantify the degree of uncertainty for less complex situations. This quantification is accomplished by assigning real numbers from 0 to 1 to the outcomes.

The student should be able to *use* most of the vocabulary introduced in the chapter. It is *not* expected that he would be able to give formal definitions. Uncertainty, equally likely, fair game, outcome, event, probability, and probability function are all terms the student should be able to use readily. For clear-cut situations, he should be able to describe a set of outcomes, assign probabilities, and to find the probability of A ∪ B, and A ∩ B where A and B are subsets of the set of outcomes.

Formal definitions are necessary for an understanding of: mutually exclusive events, complementary events, and independent events.

Chapter 12–MEASUREMENT

Purpose

The great majority of applications of mathematics, whether arithmetical, algebraic, geometric, statistical, or analytical are rooted on the idea of measurement. Historically, of course, mathematics began with need to measure and to record measurements of various kinds. In this chapter, we develop basic ideas of the measurement of length and of angles, based on the notion of congruence from Chapter 8. We deal both with theoretical considerations and properties of measures as well as with the practical problems of obtaining, reporting, and interpreting measurements of physical objects. These ideas will be used in Chapter 14 (Similarity) and elsewhere in the sequence. In Chapter 16, we shall develop measures of area and olume and introduce the idea of indirect meaurement.

Objectives

The student should understand that any segment can be used as a unit segment. He should be familiar with the standard units of length in the British and metric systems and be able to convert from any unit to any other if he is given the conversion function. He should not be expected to memorize the conversion functions.

He should know and be able to use the formulas for calculating the perimeter of a polygon and the circumference of a circle.

He should understand the definition of a degree and the terms "acute angle," "right angle," and "obtuse angle" although he may not be able to define them carefully. He should know that the sum of the measure in degrees of the angles of any triangle is 180.

Most importantly he should understand that physical measurements are approximate and have some understanding of the reason for giving the precision of a measurement. He should understand how to use a protractor to measure angles in degrees.

Comment

Include the objective of being able to convert within a particular system from one measurement to another; for example, a particular measurement from feet to inches, etc.

Chapter 13—PERPENDICULARS AND PARALLELS (I)

Purpose

The theory of perpendiculars and parallels is not only basic for the study of Euclidean geometry, but in fact, most practical applications of geometry involve this theory. In particular, important results concerning rectangles and parallelograms are developed in this chapter. From the theoretical point of view, work on parallels and perpendiculars offers more practice for the student in understanding and designing proofs. Finally, the theorems of this chapter are essential for further geometric study such as that presented in Chapters 14 and 17.

Objectives

The student should understand the content and some consequences of the theorems listed in the chapter summary although he may not be able to state each theorem precisely. In particular, he should understand several ways of deciding if two lines are parallel, be able to identify alternate interior and corresponding angles, be able to define "rectangle," "rhombus," and "parallelogram," and know properties of and relationships between these quadrilaterals. He should be able to construct parallel and perpendicular lines, a rectangle, a square, a rhombus (from Chapter 8), and a parallelogram. He should learn more about proving theorems, be able to supply reasons or statements in proofs, and be able to construct a simple proof when given the method to follow.

He should be aware that the theorem that the sum of the degree measures of the angles of any triangle is 180 depends upon the Parallel Postulate and should be able to use this theorem. He should be able to follow the use of parallels in the Eratosthenes derivation of the earth's circumference. The student should not be expected to produce original proofs (unless they are *very* simple and straightforward) or to write precise statements of complicated theorems.

Comment

Penultimate sentence, first paragraph of *Objectives:* ". . . and a parallelogram, with possibly a variety of tools." Ultimate sentence: ". . . supply reasons or statements in geometric reasoning (or arguments)."

Chapter 14—SIMILARITY

Purpose

Most students have a strong intuitive recognition that many objects appear to have the "same shape." The purpose of this chapter is to examine the mathematical basis for this property of "same shapeness." Specifically we reduce it to the congruence of corresponding angles, discussed in Chapter 8 (Congruence), and the equality of ratios of lengths of corresponding line segments. The measurement of length and of angles was developed in Chapter 12 (Measurement). The chapter draws on what the student has learned about parallel lines and transversals in Chapter 13. An attempt has been made to keep in the foreground some of the many applications of similarity; it is the basis of practically all surveying. Use is made of similarity to obtain a particularly easy proof of the Pythagorean Theorem. An introduction to trigonometric functions at the end of the chapter follows naturally from the study of similarity.

Objectives

By the end of the chapter, the student should be familiar with the conditions for similarity of triangles. For example, he should recognize that two triangles are similar if each has a 50° angle and a 75° angle. It is expected that he should be able to solve simple problems involving similar triangles. He should understand the statement of the Pythagorean Theorem and know how to apply it to problems involving right triangles, such as finding the length of one side of a right triangle when the lengths of the other two sides are known. As for the three trigonometric functions, the student should remember how they are defined in terms of adjacent side, opposite side, and hypotenuse, and be able to use these functions (with help of the table of values) to solve problems involving right triangles.

Chapter 15—THE REAL NUMBERS

Purpose

From the time of the Pythagoreans to the present day, no topic in mathematics has played such an important role as the understanding of the real number system. The concepts involved are difficult and challenging. From Chapters 7, 10, and 12, we recall that there are points on the number line that cannot be identified with rational numbers. In this chapter, we develop the idea that we may "complete" the number line by introducing irrational numbers. Many irrational numbers arise from geometric considerations such as the length of the diagonal of the unit square. Therefore, we build on the material of the earlier chapters on geometry. The question of rational approximations is discussed, with emphasis on square roots.

Objectives

An objective of this chapter is to "complete" the numbers on the number line by introducing the idea of irrational numbers. Geometric lengths exist as real "lengths" even though the set of rational numbers is not adequate to specify the exact lengths of many segments. The concepts of length and distance naturally lead to square roots and we include as an objective some facility in operating with square roots, manipulating radicals, and obtaining rational approximations. Other objectives include better understanding of the structure of the number systems through the extensions from the natural numbers to the real numbers.

The following is a suggestion for further reading on the subject for teachers and students. These are volumes in the series of The New Mathematical Library, SMSG:

Numbers: Rational and Irrational by Ivan Niven

The Lore of Large Numbers by Philip Davis

Uses of Infinity by Leo Zippin

Chapter 16—SECOND LOOK AT MEASUREMENT

Purpose

This chapter continues the study of measurement begun in Chapter 12. It also relies on the discussion of congruence in Chapter 8, decimal representation in Chapter 10, properties of rectangles in Chapter 13, the Pythagorean Theorem in Chapter 14, and computation with real numbers in Chapter 15. Many of the formulas for area and volume are developed by the students in exercises. The discussion of precision in Chapter 12 is extended to computation with measurements (approximate numbers) and the hazards involved. Other kinds of direct measurement and some examples of indirect measurements that students will encounter in such courses as science and shop are introduced.

Objectives

The student should know and be able to use the formulas for the area of a rectangle, triangle, parallelogram, trapezoid, and circle. He should understand and be able to use the formulas for the area of a rhombus (in terms of its diagonals), a ring, and a sector of a circle if he is given the formula or is reminded of its derivation. He should be able to derive similar formulas if they are presented to him in a step-by-step approach.

He should know and be able to use the formulas for the volume of a box and a sphere. He should know that the volume of many solids, including triangular prisms and cylinders, is the product of the area of the base and the height of he solid. He should be familiar with standard nits of area and volume. He should be able to se the definitions of surface area of each solid ut should not be expected to memorize the for- ulas.

He should be able to make generalizations bout how area or volume changes when the near dimensions are doubled, tripled, or halved. e should be aware that error can be compounded hen he computes with numbers that are ap- oximations. He should understand that there e other kinds of measurements (indirect) which not have the properties that length, area, and ume do.

Most importantly, this chapter should contribute to his understanding of mathematical derivations and of how mathematics is applied to the real world.

Comments

Additional objective: Understanding of units in the sense that the particular kind of unit used for measurements is important; that is, for instance, square inches are used for areas expressed in inches; that conversions may be made, but inches are not mixed up with feet in the computations.

Second sentence in first paragraph, *Objectives,* suggest changing to: He should understand and be able to apply these formulas to other areas constructed from other basic areas.

Chapter 17–PERPENDICULARS AND PARALLELS (II)

Purpose

A major purpose of this chapter is to derive methods of arriving at conjectures, to recognize that conjectures require proof, and to prove some of them. The methods of arriving at conjectures include paper folding and asking whether theorems about parallel and perpendicular lines in a plane can be generalized to space. Another purpose of the last three sections is to develop the student's ability to visualize 3-dimensional figures and relations in space. This chapter builds on and extends the geometry developed in Chapters 1, 8, and 13. Chapters 13 and 17 together contain the usual theorems on parallels and perpendiculars from Euclidean geometry which have many important applications in the real world.

Objectives

The student should understand the meaning of such terms as "altitude," "angle bisector," "perpendicular bisector," "median," "tangent to a circle," "dihedral angle," "plane angle of a dihedral angle," and "mutually perpendicular planes." He should be able to construct by paper folding the various lines associated with a triangle, such as the medians, and see how compass and straightedge constructions are related to paper folding "constructions." He should be able to construct a tangent to a circle at a point of tangency or from an external point and to construct a rectangle having a given side and a given diagonal (provided the diagonal is longer than the side). He should be able to prove simple theorems about medians, etc., of a triangle, use the hypotenuse-leg theorem, understand and explain the tangent constructions, and know that any angle inscribed in a semicircle is a right angle. He should be able to visualize such figures as parallel, skew, and perpendicular lines in space, parallel and perpendicular planes, a line perpendicular to or parallel to a plane, and the set of points equidistant from the end-points of a line segment in space. He should be able to think about whether a theorem in 2-dimensional space can be generalized to a theorem in 3-dimensional space.

Chapter 18–COORDINATE GEOMETRY

Purpose

This chapter brings together the previous development of geometry, of functions, and the algebraic development of number systems, and lays the foundation for the study of a mathematical system called analytical geometry. Therefore, this chapter is a key chapter in the grade 7 through grade 9 sequence.

The purposes of this chapter are

(1) to introduce the students to the way in which certain geometric ideas may be formulated and expressed algebraically;

(2) to indicate the power of these algebraic formulations in analyzing some mathematical ideas.

The following background, developed in earlier chapters, is assumed:

(1) familiarity with coordinates of points on a line and coordinates of points in a plane and how to plot them;

(2) some understanding of the function concept;

(3) ability to graph linear functions;

(4) understanding of perpendicularity and parallelism, and of definitions and properties of certain geometric figures;

(5) ability to transform equations from one form to another.

Objectives

The student should be able to

(1) use meaningfully the distance and midpoint formulas in 1-, 2-, and 3-dimensional coordinate systems,

(2) describe algebraically, using set builder notation, certain geometric figures in each of the coordinate systems, and conversely, to describe geometrically the solution sets of

points of certain equations,

(3) determine slopes of lines and to relate slope to parallel and perpendicular lines,

(4) develop coordinate proofs of certain properties of some geometric figures.

Chapter 19–PROBLEM SOLVING

Purpose

All of mathematics is concerned with problem solving. Some problems are theoretical and many are "practical." Problems of various types occur, obviously, throughout secondary school mathematics. In each chapter, we have been concerned with specific techniques developed in the chapter. There are, however, certain general strategies and methods that are useful in all types of problems. The major purposes of Chapter 19 are to direct the student's attention to some of these strategies and techniques. The formulation of a mathematical model for a given "real" problem, the analysis of this model, and the interpretation of the analysis in terms of the original problem are stressed. Hopefully, the practice provided by the examples and exercises of this chapter will be reinforced throughout the student's pursuit of his mathematical studies.

Objectives

(a) To provide the student with a variety of strategies for problem solving.

(b) To develop some flexibility in the student's approach to problem solving.

(c) To develop techniques for using geometric representations as a way of developing new information about a given situation.

(d) To develop some skill in using a tabular arrangement, of given and derived information, to help solve problems.

(e) To develop, for the student, a better understanding of a problem by teaching him to make numerical estimates and testing them in the actual problem.

Chapter 20–SOLUTION SETS OF MATHEMATICAL SENTENCES

Purpose

If there is one single skill which is the most important for the applications of mathematics, it is the ability to solve simple algebraic equations (and inequalities). This ability is so fundamental that it occurs in almost every chapter of Secondary School Mathematics. In Chapter 9, we began the formal development of techniques of finding solution sets. Chapter 20 reviews and extends these techniques. The stress is on the idea of deriving, from a given open sentence, a chain of equivalent sentences using the properties of the real number system. The final sentence in the chain should be one whose solution set is obvious. Incorporated in this chapter is some practice on simple manipulation with polynomials and rational fractions (although these terms are not used). While the emphasis is on linear equations and inequalities, some work on quadratic polynomials lays the foundation for Chapter 23 (Quadratic Functions). In addition, the skills developed in Chapter 20 are, of course, fundamental for Chapter 25 (Systems of Sentences in Two Variables).

Objectives

We are concerned with equations and inequalities which, after simplification, involve only the first power of a single variable. The student should be able to solve such equations and inequalities efficiently and accurately. It is also expected that the student will be able to handle certain types of factoring. He should be able to solve equations which are written as a product of linear factors equal to 0.

Additionally, the student will gain experience with "translating" verbal problems into mathematical language and with solving the resulting sentences.

Comment

In the first sentence of the *Objectives* statement, "simplification" should be replaced by "some analysis."

Chapter 21—RIGID MOTIONS AND VECTORS

Purpose

Euclidean geometry is, in essence, the study of the properties of figures which are preserved under a certain set of transformations. These transformations are those which are discussed in this chapter—translations, rotations, and reflections. Thus, Chapter 21 is of primary importance for an understanding of Euclidean geometry. In Chapter 8 (Congruence), the student had experience in using a tracing sheet. The process of moving a racing sheet from one position to another suggests the general idea of a rigid motion. We now develop the idea more carefully. The important notion of symmetry rises from considering reflecions. Coordinate geometry (Chapter 18) is used to express certain rigid motions in terms of a "change of coordinates." This work is exploited in Chapter 23 (Quadratic Functions). Translations are representable by vectors, so a brief introduction to two-dimensional vectors is presented. Applications are made to some of the usual problems involving navigation, equilibrium of forces, as well as to geometric proofs.

Objectives

The important items in this chapter are much more the *ideas* than any particular operational skills.

(a) *Understandings expected*

The meaning of a rigid motion, in particular of translation, reflection, and rotation.

The meaning of symmetry and axis of symmetry.

The composition of two rigid motions.

That usually the composition of two rigid motions depends on the order in which they are taken.

That for two translations the composition does not depend on the order.

That a translation can be described by an arrow.

That a reflection is determined by its axis.

That the coordinate plane can often be used to describe rigid motions.

The association of vectors in a plane with translations.

The meaning of a vector sum.

(b) *Performance expected*

If a rigid motion is suitably described, to find the image of a given figure under this motion.

For a given figure to decide if a given line is an axis of symmetry.

To find all axes of symmetry for a simple figure.

In simple cases, given two congruent figures, to describe explicitly a sequence of one or more translations, reflections, and rotations mapping one figure onto the other.

Given a point in the coordinate place and its image under a translation, to find the image of any other point. Given a point in the coordinate plane, to find its image under reflection in the X-axis or in the Y-axis.

To solve simple problems involving velocities and forces making use of vector methods.

Use vectors to prove certain geometric theorems.

Chapter 22–COMPUTERS AND PROGRAMMING

Purpose

There is no need to stress the increasing importance of computers in modern society. Building heavily on the flow charting work of Chapter 3, we introduce the student to the elements of a digital computer, an assembly language, and to the basic language. Use is made of algorithms developed in earlier chapters to provide illustrative material. The chapter should provide a minimal understanding of the capacities and limitations of computers. Students who wish to learn more about computers and computer language should find this chapter a useful introduction to programming.

Objectives

Students should gain a better understanding of how computers work and the role of programmers in making computers work by studying this chapter. If they have the opportunity to run some of their programs, they will gain much greater insight into the challenges of programming. By analyzing how a computer evaluates expressions, students learn more about mathematical symbolism and algorithms. Such algorithms as "divide and average" and the solution of the general linear equation in one variable should become more meaningful. Programming may motivate students to continue studying mathematics.

Chapter 23–QUADRATIC FUNCTIONS

Purpose

This chapter provides a systematic analysis of quadratic functions and a formalization of methods of solving quadratic equations. Extensive use is made of previously studied concepts. In particular, we build upon the study of linear functions, upon coordinate geometry and upon the early ideas about finding the solution sets of mathematical sentences. The skills developed in this chapter are useful in the later study of exponential and logarithmic functions (Chapter 26).

Objectives

The student should be able to

(a) identify the essential characteristics of the graph of a quadratic function when the function is expressed in standard form;

(b) solve quadratic equations by the method of completing the square;

(c) state the necessary restrictions on the variable in equations involving square roots, absolute values, and rational expressions which lead to quadratic equations, and to solve these equations.

Comment

The Board is not convinced that everyone needs this much exposure to quadratic functions.

Chapter 24—STATISTICS

Purpose

The increasing reliance on statistical methods in the decision making process in many branches of human activity makes it essential that the educated citizen have some familiarity with statistical techniques. In this chapter we attempt to present some elementary ideas about organizing numerical data and of reporting summary descriptions of the data. We concentrate on the use of the mean and the standard deviation as two numbers which summarize information about a collection of numerical data.

Statistical methods are most often used to reach decisions or to make predictions. Such decisions and predictions involve probabilistic considerations. In Chapter 28 we shall bring together the methods of Chapter 24 and the probability theory of Chapter 11.

Objectives

We expect the student to understand how to construct and understand simple tables and graphs. In a similar way, he should understand that the mean and standard deviation are useful in reporting a "summary" of information about a distribution. The level of expected competency in calculation is shown by the nature of the exercises. The material on the use of the normal curve (Section 24–7) is preliminary. It will be reviewed and reemphasized in Chapter 28.

Chapter 25—SYSTEMS OF SENTENCES IN TWO VARIABLES

Purpose

All the machinery of problem formulation, construction of mathematical models, solution of equations and inequalities, and coordinate geometry is brought together in this chapter. Challenging problem situations are presented whose solutions involve the techniques of linear programning. In order to solve such problems, the student led to consider systems of linear equations and nequalities. Graphical and algebraic methods of olution are presented. The algebraic argument is ased on the idea of equivalent systems. The simi- r idea of equivalent sentences has been care- lly developed in Chapters 9 and 20.

bjectives

The student should become capable of solving system of linear equations

$$Ax + By + C = 0$$
$$Dx + Ey + F = 0.$$

It is expected that the student should be able to nderstand and to use both the algebraic methods of "elimination" and "substitution") as well as raphical methods.

Similarly, the student should understand that ne solution set of a linear inequality such as $x + By + C < 0$ consists of the points in a alf-plane. He should be able to graph the solu- on set of system of two or more linear inequali- es.

We anticipate that the student will be able to se these techniques to arrive at the solution of an lementary linear programming problem.

hapter 26—EXPONENTS AND OGARITHMS

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A major purpose of this chapter is to develop nanipulatory and computational skills. Using in- egers as exponents leads to the useful scientific otation. The introduction of rational numbers as xponents is of assistance in working with radicals.

Simple arithmetic can be done by "longhand" nethods; extensive calculations can be performed fficiently by computers. Logarithms are the major ool in the middle ground for computations that re too laborious for longhand and too simple to ustify expensive computer time.

The development of the material in this chapter of interest in itself. The student discovers that ne basic idea (positive integral exponents) can ad, step-by-step, to the much more sophisticated ncepts of exponential and logarithmic func- ons.

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This chapter provides the student's first expo- re to a large number of new ideas, symbols, and hniques. It is too much to expect complete stery of them all. If the student can handle

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most of the exercises, this is sufficient. Specifical
we hope the student acquires familiarity with

(1) the definition of a^x for x an integer,
(2) the interpretation of $a^{p/q}$ as $\sqrt[q]{a^p}$,
(3) $\exp_a$ and $\log_a$ as functions,
(4) common logarithms.

This familiarity should be sufficient so that t
student is able to

(1) manipulate freely with integral exponen
including the ability to use scientific not
tion;
(2) simplify ordinary expressions involving r
tional exponents or radicals;
(3) use a table of mantissas with sufficient ski
to apply logarithmic techniques to arit
metic computations.

Only a beginner's level of skill is anticipate
For example, it is not planned that the student
able to interpolate. It is expected that he wou
have to proceed very slowly in "simplifying" e
tremely messy expressions. Once again, the hope
for degree of skill in manipulation is indicated
the exercises.

The section on the slide rule is strictly intr
ductory and no practical use of a slide rule is d
veloped.

Comment

The second sentence of the second paragrap
provides an illustration of the speed with whic
technology progresses. Less than a year after it w
written, small, inexpensive, battery powered ele
tronic calculators were available to handle th
"middle ground for computations that are to
laborious for longhand and too simple to justi
expensive computer time."

Chapter 27–LOGIC

Purpose

Logical thinking is inherent in all of math
matics. From this point of view, a careful discu
sion of logical principles might have come earli
in the sequence. Until the student acquires a goo
deal of mathematical experience, however, it
difficult to illustrate the need for carefully org
nized methods of deduction. We can in this cha
ter discuss the logical framework which underli
the mathematical proofs to which he has bee
exposed. As illustrations, we re-examine certai
proofs used in preceding chapters. Both dire
proof and proof by contradiction are discussed i

me detail. The methods of proof discussed in
is chapter will be refined and utilized in future
urses.

bjectives

At the end of this chapter, the student should
nderstand and be able to correctly use the terms:

> Simple statement, compound statement, negation, conjunction, disjunction, conditional, biconditional, logically equivalent, converse, contrapositive, direct proof, proof by contradiction, quantifier.

e should be able to symbolize statements, con-
ruct truth tables and use them to determine
gical equivalence. He should be able to con-
ruct a simple direct proof in either logical or
athematical form and complete a proof by con-
adiction. He should be able to state the nega-
on of a quantified statement, using Euler dia-
ams as an aid if he wishes.

omment

Under *Purpose*, include the notion that logical
inking is useful outside mathematics. Emphasize
e necessity for precision in use of language.

hapter 28—PROBABILITY AND TATISTICS

rpose

Our major goal is to lead the student to some
derstanding of statistical results which are so
ten reported. We frequently encounter such
atements as "the students of our school per-
rmed significantly better than the average of all
udents in the school system," "the opinion poll
2000 voters shows that candidate A will be
ected," or "the new medicine has no significant
fect on the recovery rate of patients." In this
apter we discuss the mathematical methods and
guments that are used in reaching conclusions
this kind. Our tools, of course, are the materials
Chapters 11 and 24.

bjectives

One major emphasis is that statistical conclu-
ons, based on samples, are probabilistic in na-
re. There is enough material developed in the
apter so that the student should be able to
nderstand the *meaning* of statistical statements
at he will encounter in the future. In order to
quire such understanding, the student should
able to follow the general arguments of the
apter. The binomial case is emphasized, since

this provides the simplest example of a situatic
in which a hypothesis may be tested by samplin
After finishing the chapter, the student should
able to state a simple hypothesis and to unde
stand that experimental evidence could lead
rejection or acceptance of the hypothesis. Ma
students may eventually forget the details (f
example, that $\sigma_s = \sqrt{npq}$ for a binomial distrib
tion), but the experiences should enable them
later years to exercise intelligent judgment
evaluating assertions based on statistical evidenc

In particular, he will have sufficient backgrou
to move later into a more traditional course
introductory statistics and probability given p
haps at the senior high school or college freshm
level. Another possibility is for the interested st
dent to study the programmed course given
Introduction to Probability, School Mathemati
Study Group, Stanford University, 1965. Part I
this text offers a careful treatment of independe
events, based on the important notion of con
tional probability.

General Comments

1. Since the curriculum is for "all" students, t
objectives of exercising, strengthening, and
viewing previously learned materials need to
kept in mind.
2. The objectives of estimating and approxim
ing have not been mentioned.
3. A number of the chapter statements omit me
tioning the skill objectives which are built in
the text.
4. An overall objective is *extending* from o
mathematical system to a larger one.